# Potential Drop and Ionic Flux in Desalting Electrodialysis Units

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ABSTRACT. In this paper, the differential equations governing the ionic flux, current intensity and potential drop for an electrodialysis (ED) unit are derived in terms of ionic concentration, ionic diffusion coefficient in both water and membranes, ionic electric transfer numbers and membrane properties. These equations are solved to obtain the unit potential difference together with the ionic flux of counter and co-ions through the membranes. The power required for the unit is obtained by multiplying the potential drop by the integral of the current intensity along the flow passage. The minimum power required is calculated and compared with an approximate analysis for the ED unit at different working conditions. The ratio between minimum power and approximate power ranges from 62-46 percent at C<sub>f</sub> of 2000 ppm to 26-19 percent at C<sub>f</sub> of 10000 ppm by using compartment thickness of 0.5-1 mm, respectively.

#### Introduction

Electrodialysis (ED) is a solute transfer process, *i.e.* dialysis, through ion exchangers type of membranes and electric potential difference as the driving force for the solute transfer process. Such process is rapidly gaining more potential in desalting brackish waters containing total dissolved solids ranging from 1000 to 10,000 ppm<sup>[1]</sup>. The first commercial ED system was used, in 1954, by ARAMCO, Saudi Arabia<sup>[2]</sup> to produce drinking water from brackish wells for oil drilling company use.

The electrodialysis system, Fig. 1, comprises a stack of ion exchange membranes terminated on each end by a DC electrode. The membranes are arranged alternatively of Cation Exchange Membranes, *e.g.*, Na<sup>+</sup>, Ca<sup>++</sup> and Mg<sup>++</sup> (CEM), and Anion Exchange Membranes, *e.g.*, Cl<sup>-</sup>, SO<sub>4</sub><sup>--</sup>, CO<sub>3</sub><sup>--</sup> (AEM). These membranes are thin plastic sheets of cross linked organic polymers with ion exchange properties,

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*e.g.*, sulfonated polystyrene-divinyl-benzene polymers. The saline water is introduced to the compartments located between each pair of membranes. Whenever a direct electric current is applied through the electrodes, the ionic constituents are removed from the feed stream in one set of alternate compartments (called the dialysates) to the other set of alternate compartments (called the concentrates).



FIG. 1. An electrodialysis unit (schematic drawing).

Electrical resistance and permselectivity are the two main parameters of importance in characterizing the ion exchange membranes. Under normal operating conditions, the minimum salinity of product water is restricted to 300-500 ppm (TDS)<sup>[3]</sup> in order to limit the power cost. In seawater applications, however, the membrane should exhibit an area of resistance of about 20 Ohm-cm<sup>2</sup>. The permselectivity for synthetic ion exchange membranes diminishes with increasing the salinity (concentration). Good membranes show a permselectivity over 0.95 in the 0.1 N range, 0.9 in the 0.5 N range and 0.85 in the 1.0 N range (seawater is normally treated as 0.6 N)<sup>[4]</sup>.

The design of a successful and economical electrodialysis system is affected by a number of parameters. These include the voltage drop across a pair of membranes and the electric power consumed in the desalting process. This paper describes, in detail, an easy to follow, method to derive the voltage drop and power consumption of an electrodialysis unit.

## **Governing Equations**

In an elctrodialysis system, ions are transferred under the influence of both the chemical potential gradient and the electrical potential gradient. The ions flux (rate

of the transported ions per unit area normal to the transport direction) is related to the driving force through the following fundamental formulae:

$$Flux = velocity \times concentration$$

The mobility (u) of an ion is defined as its velocity in the field direction divided by the field driving force. Substitution gives the flux equation as following,

 $Flux = mobility \times concentration \times driving force.$ 

So, the flux J of species i in the x-direction due to chemical potential gradient is

$$J_{i,df} = u_i C_i \frac{d\mu_i}{dx}$$
(1)

where  $\mu_i$  is the chemical potential of the species *i* and is given in terms of the activity  $a_i$  as following :

$$\mu_i = \mu_i^0 + RT \ln a_i$$

where  $\mu_i^0$  is the chemical potential at the reference state. Also the activity  $a_i = \gamma_i C_i$ , where  $\gamma_i$  is called the activity coefficient. For the case of dilute solutions,  $\gamma_i \simeq 1$ . Hence,

$$J_{i,df} \simeq - u_i RT \frac{dC_i}{dx}$$
(2)

comparison with the Fick's law of mass diffusion give the diffusion coefficient  $D_i$  equals to  $u_i RT$ .

The force acting on ions in an electrical field and causes their movement is the electric field intensity (E) defined as follows,

$$E = -\frac{d\phi}{dx}$$

where  $\phi$  is the electrical potential. The electrical potential difference between two points is the work required to transfer a unit charge between them, thus, the flux due to electric current is equal to

$$J_{i,e} = - u_i C_i Z_i F \frac{d\phi}{dx}$$
(3)

where  $Z_i$  is the valence of the ion, F is the Faraday number (*i.e.*, the elementary charge multiplied by Avogadro's number N which is the number of molecules per mole). Hence, the electrical quantities carried by a mole of species i is  $Z_i eN = Z_i F$ . Then, the combined flux due to the simultaneous diffusional and electrical forces for one dimensional flow can be expressed as follows,

$$J_i = -D_i \left(\frac{dC_i}{dx} + Z_i C_i \frac{F}{RT} \frac{d\phi}{dx}\right)$$
(4)

where  $D_i = u_i RT$ 

#### Voltage Drop Across an Electrodialysis Unit

In operating the electrodialysis unit, enough electrical potential should be applied to overcome the ohmic resistance (I/K), the potential drop across the membranes, the dialysate and concentrate compartments drop due to concentration gradients, potential drop at the electrodes, and the potential drop at the membranes-liquid interfaces (Donnan potential). The first step in the analysis is to determine the potential drop across a half cell containing a cation exchange membrane, half dialysate and half concentrate compartments. Then, the potential drop across an electrodialysis unit is obtained. The considered half cell is divided as shown in Fig. 2 into five regions I, II, III, IV and V. For any of these regions, the concentration distribution is assumed, and equation (4) is then integrated in order to obtain the potential drop for that region. Some simplifications are assumed, and justified, in the integration procedure. These are pointed out whenever appropriate.



FIG. 2. Potential drop in an ED cell.

## 1. Region (I,V) The Concentrate and Dialysate Compartments

For simplification, a monovalent electrolyte solution, *e.g.*, NaCl, will be considered. Consequently, the condition of electric neutrality is

$$C_k Z_k + C_a Z_a = 0, \qquad Z_k C_k = |Z_a| C_a$$

The electric current intensity through a solution due to electric field is the total

number of the unit charges passing through a plane. Then, for the simple case of one electrolyte solution, the electric current intensity will be equal to;

$$I = F(Z_k J_k + Z_a J_a) \tag{5}$$

where the subscripts a and k refer to the anion and cation species, respectively. Substitution of equation (5) in the Nernest-Plank equation (4) gives the relation between I and  $\phi$  by

$$\frac{d\phi}{dx} = -\frac{I}{K} - \frac{RT}{F} \left( \frac{t_k}{C_k} \frac{dC_k}{dx} + \frac{t_a}{C_a} \frac{dC_a}{dx} \right)$$
(6)

where K is specific electric conductance and equals to

$$K = \frac{F^2}{RT} \left( D_k C_k Z_k^2 + D_a C_a Z_a^2 \right)$$
(7)

where  $t_k$  and  $t_a$  are the cation and anion transferrence number, respectively, and equal to

$$t_{k} = \frac{D_{k}Z_{k}C_{k}}{D_{k}Z_{k}^{2}C_{k} + D_{a}Z_{a}^{2}C_{a}} \text{ and } t_{a} = \frac{D_{a}Z_{a}C_{a}}{D_{k}Z_{k}^{2}C_{k} + D_{a}Z_{a}^{2}C_{a}}$$
(8)

Introducing the condition for electric neutrality into equations (7), (8) gives expressions for  $t_k$ ,  $t_a$ , and K as

$$K = \frac{F^2}{RT} Z_k C_k \left( D_k Z_k + D_a \mid Z_a \mid \right) = Z_k C_k A$$

where

$$\Lambda = \frac{F^2}{RT} (D_k Z_k + D_a Z_a)$$
  
$$t_k = \frac{D_k}{D_k Z_k + D_a Z_a} , \text{ and } t_a = \frac{-D_a}{D_k Z_k + D_a Z_a}$$

thus,  $t_k$  and  $t_a$  do not depend on concentration and are considered to have constant values.

In regions I and V, in Fig. 2,  $Z_k C_k = Z_a C_a$  and dC/dx = 0. The integration of equation (6) within the bulk of flow for one half of the compartment (from center line to boundary layer region) gives:

$$\phi_1 - \phi_0 = -l (b/2 - \delta_1)/K_1$$
(9)

$$\phi_3 - \phi_2 = -I(b/2 - \delta_2) / K_2 \tag{10}$$

For sodium chloride case,  $C_1 = C_{a_1} = C_{k_1}$  and  $Z_k = |Z_a| = 1$ , therefore  $K_1 = F^2 C_1 (D_k + D_a) / RT$  and  $K_2 = F^2 C_2 (D_k + D_a) / RT$ 

#### 2. Concentration Boundary Layers Regions II, IV

In order to simplify the integration of equation (6) in these concentration boundary layers, the concentration is assumed to have a linear relationship with the distance x. This gives ;

$$\frac{C_k - C_{k_1}}{C_{k_1} - C_{k_1}} = \frac{x}{\delta_1}$$
(11)

Substitution from equation (11) into equation (6) and performing integration yields;

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$$\phi'_{1} - \phi_{1} = -\frac{I\delta_{1}}{K'_{1} - K_{1}} \ln K'_{1}/K_{1}$$
$$-\frac{RT}{F} (t_{k} - |t_{a}|) \ln (C_{k'_{1}}/C_{k_{1}})$$
(12)

$$\phi_{2} - \phi_{2}' = -\frac{I\delta_{2}}{K_{2} - K_{2}'} \ln (K_{2}/K_{2}') - \frac{RT}{F} (t_{k} - |t_{a}|) \ln (\frac{C_{K_{2}}}{C_{K_{2}'}})$$
(13)

where  $K_1, K'_1, K_2 \& K'_2$  are the values of the conductance given by equation (7) at the concentration  $C_{K_1}, C_{k'_1}, C_{k'_2}$ , and  $C_{k'_2}$ , respectively.

# 3. Membrane-Phase (Region III)

The cation exchange membrane, for example, is permeable to cations and impermeable to anions due to the existence of fixed negative ions and mobile cations of concentration higher than those in the solution. At the same time, the solution has higher concentration of mobile anions than that of the membrane. These concentration differences favour the transportation of some cations from the membrane to the solution and some anions in the reversed direction. The migration of these ions creates negative charges in the membrane and positive charges in the solution at the membrane solution interface. Accordingly, concentration discontinuity and electric potential difference are developed at the liquid membrane interface. Hence, an equal potential difference (called Donnan potential) is created to drive the cations back to membrane and the anion back to the solution and, thus, restoring equilibrium conditions. This is reached when the tendency of the ions to level out the existed concentration differences is balanced by the action of the Donnan potential. Indeed, equilibrium is satisfied when the electro-chemical potentials on both sides of the interface are equal, for species *i* at the interface, this gives;

$$\mu_i^{\rm o} + RT \ln \gamma_1' C_1' + Z_i F \phi_1' = \mu_i^{\rm o} + RT \ln \overline{\gamma}_1 \overline{C}_1 + Z_i F \overline{\phi}_1$$
(14)

(the superscript bar indicates the membrane phase). Equation (14) is valid for all ionic species, and for NaCl

$$E_{\text{Donnan}} = \phi_1' - \overline{\phi}_1 = \frac{RT}{F} \ln \frac{\gamma_{K_1'} C_{K_1'}}{\overline{\gamma}_{K_1} \overline{C}_{k_1}}$$

$$= \frac{RT}{F} \ln \frac{\overline{\gamma}_{a_1} \, \overline{C}_{a_1}}{\gamma_{a_1'} \, C_{a_1'}} \tag{15}$$

The ratio of the mean activity coefficient in the phase is defined by

$$\lambda = \frac{\gamma_k}{\overline{\gamma}_k} = \frac{\gamma_a}{\overline{\gamma}_a} \tag{16}$$

The ratio  $\lambda$  is considered constant and uniform throughout the membrane phase, similarly,

$$\bar{\phi}_2 - \phi'_2 = -\frac{RT}{F} \ln \frac{\bar{C}_{k_2}}{C_{k'_2}} = -\frac{RT}{F} \ln \frac{C_{a'_2}}{\bar{C}_{a_2}}$$
(17)

However, the potential drop in the membrane phase itself is tedious to obtain. Therefore, only the simple case of monovalent solute, e.g. NaCl, is considered. For the case of cation exchange membrane of X ion-exchange capacity, the electroneutrality condition requires that

$$Z_k \bar{C}_k + Z_a \bar{C}_a = wX \tag{18}$$

where w = 1 for cation exchange membrane and w = -1 for anion exchange membrane. For simplicity, assume a linear relation between the ionic concentration in the membrane and the distance x across it, for  $Z_k = Z_a = 1$ , and for a thickness of the membrane =  $l_k$ , lead to the following;

$$\bar{C}_{k} = \bar{C}_{k_{l}} + \frac{x}{l_{k}} (\bar{C}_{k_{2}} - \bar{C}_{k_{1}})$$
(19)

The use of equations (18) and (19) in integrating equation (6) gives

$$\bar{\phi}_{2} - \bar{\phi}_{1} = -\frac{RT}{F(\bar{D}_{k} + \bar{D}_{a})} \left[ \frac{I}{F} \frac{l_{k}}{(\bar{C}_{k_{2}} - \bar{C}_{k_{1}})} + (\bar{D}_{k} - \bar{D}_{a}) \right] \ln \frac{(\bar{D}_{k} + \bar{D}_{a})}{(\bar{D}_{k} + \bar{D}_{a})} \frac{\bar{C}_{k_{2}} - \bar{D}_{a}X}{\bar{C}_{k_{1}} - \bar{D}_{a}X}$$
(20)

The use of equations (15), (16) and (18) to relate the concentration in the membrane and that in the solution gives, *e.g.*,  $\overline{C}_{k_1}$  and  $C_{k_1}$ 

$$\bar{C}_{k_1} \approx \frac{X + \sqrt{X^2 + 4\lambda^2 C_{k_1}^2}}{2}$$
(21)

and for high capacity membrane ( $\lambda C_{k'} \ll X$ ), the value of  $\overline{\phi}_2 - \overline{\phi}_1$  is,

$$\bar{\phi}_{2} - \bar{\phi}_{1} = -\frac{RT}{F\bar{D}_{k}} \left[ (\bar{D}_{k} - \bar{D}_{a}) + \frac{I}{F} \frac{l_{k}}{\lambda^{2} (C_{k_{2}}^{2} - C_{k_{1}}^{2})/X} \right]$$

$$\frac{\lambda^{2}}{X^{2}} (C_{k_{2}}^{2} - C_{k_{1}}^{2}) \qquad (22)$$

The membrane potential,  $E_m$ , is the sum of Donnan potential on both sides of the membrane and the diffusional potential (*i.e.*  $\overline{\phi}_2 - \overline{\phi}_1$  at I = 0) can be derived for high capacity membranes ( $\lambda C_{k'} \ll X$ ) as follows;

$$E_m = -\frac{RT}{F} \left[ \ln \frac{C_{k_2}}{C_{k_1}} - \frac{\lambda^2}{X^2} \frac{\bar{D}_a}{\bar{D}_k} \left( C_{k_2}^2 - C_{k_1}^2 \right) \right]$$
(23)

#### 4. Potential Across a Cell

The potential drop across a cell,  $E_c$ , is the sum of the potential drop across its half that contains the cation exchange membrane [obtained by adding Eqs (9), (10), (12), (13) and (23)] and its other half which contains the anion exchange membrane. By assuming similar concentration distribution in the other half cell that contains the anion exchange,  $E_c$  is expressed by;

$$E_{c} = I \left[ \frac{b - 2\delta_{1}}{AC_{1}} + \frac{b - 2\delta_{2}}{AC_{2}} + \frac{2\delta_{1} \ln C_{1}'/C_{1}}{A(C_{1}' - C_{1})} + \frac{2\delta_{2}}{A(C_{2} - C_{2}')} \ln \frac{C_{2}}{C_{2}'} + R_{km} + R_{am} \right] + \frac{RT}{F} \left( \overline{t}_{k} - |\overline{t}_{a}| \right)_{km} \ln \frac{C_{2}'}{C_{1}'} + \frac{2RT}{F} \left( t_{k} - |t_{a}| \right) \ln \frac{C_{2}C_{1}'}{C_{1}C_{2}'} + \frac{RT}{F} \left( \overline{t}_{a} - |\overline{t}_{k}| \right)_{am} \ln \frac{C_{2}'}{C_{1}'}$$

$$(24)$$

where

$$\Lambda = \frac{F^2}{RT} (D_k Z_k + D_a | Z_a |)$$

$$R_{km} = \frac{l_k}{(K_{lm})_{km}} \text{ and } R_{am} = \frac{l_a}{(K_{lm})_{am}}$$

$$(K_{lm})_{km} = \frac{\overline{K}_2 - \overline{K}_1}{\ln \overline{K}_2/\overline{K}_1}, \ \overline{K} = \frac{F^2}{RT} (D_k Z_k^2 \overline{C}_k + D_a Z_a^2 \overline{C}_a)$$

The subscript *lm* means the logarithmic mean difference, and the subscripts *am*, and *km* denote the anion and cation exchange membrane, respectively,  $l_k$  and  $l_a$  are the thickness of the cation and anion exchange membranes, respectively.

#### 5. Power Consumption

The power consumed by a cell, is obtained by multiplying, the potential across the cell,  $E_c$  by the current density passing through an elemental area and then by performing integration over the whole cell area, thus,

$$P_c = \int_{A=0}^{A_m} E_c \, I dA \tag{25}$$

where  $A_m$  is the membrane area. It is clear that, the fluid concentration increases along the concentrate compartment, while decreases along the dialysate compartment. Consequently, the current density, which depends on the resistance which is, in turn, a function of the density, is expected to vary along the flow water path. The theoretical minimum potential drop occurs at zero ohmic resistance and no concentration gradient. The zero ohmic resistance cancels the first term of equation (24) and the absence of concentration boundary layer gives  $C_1 = C'_1$  and  $C_2 = C'_2$ . Hence, equation (24) is reduced and will be;

$$E_{c,min} = \frac{RT}{F} \left( \overline{t}_k + |\overline{t}_a^k| - |\overline{t}_a| - \overline{t}_k^a \right) \ln \frac{C_2}{C_1}$$
(26)

where  $\overline{t}_k^a$  and  $\overline{t}_a^k$  are the transport number of anions through the CEM and the cations through the AEM, respectively.  $E_{c,min}$  is the potential drop which exists due to the concentration difference between the dialysate and the concentrate compartments. The actual potential drop is related to  $E_{c,min}$  as following;

$$E_c = E_{c,min} + R_a I_{mean} \tag{27}$$

where  $R_a$  is the apparent ohmic resistance of the cell, and  $I_{mean}$  is the average current density applied for an electrodialysis stage of *n* cells. The voltage drop per stage will be;

$$E_{stage} = n E_{c}$$
$$= n E_{c,min} + n R_{a} I_{mean}$$
(28)

And the minimum power consumed by the cell will be;

$$\int_{A=0}^{A_m} E_{c,min} \ IdA = P_{c,min}$$
(29)

The situation can be simplified by assuming a linear relation between the concentrations in the flow compartments and the coordinate y along the path of the water flow. This coordinate is measured starting from the membrane inlet and along its length to its full height H (see Fig. 3). The y-concentration relationship is expressed as following;

$$\frac{C_F - C_1}{C_F - C_d} = \frac{y}{H} \text{ and } \frac{C_2 - C_F}{C_b - C_F} = \frac{y}{H}$$
(30)

where  $C_F$ ,  $C_1$  and  $C_d$ , are the concentrations at the inlet, at a distance y and at the exist of the dialysate compartment. Also  $C_F$ ,  $C_2$ , and  $C_b$  are the concentrations at the inlet, at a distance y and at the exit of the concentrate compartment. If the dialysate outlet flow rate from n cells is  $M_d$  in kg/s, then the quantity of salt removed from one

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cell is,

$$\frac{M_d}{n} (C_F - C_d) = \frac{\eta}{F} \int_0^H I dy$$
(31)

where  $\eta$  is the Coloum efficiency. Substitution of equation (26) into equation (29) and using equations (30) and (31) gives, with 100 percent Coloum efficiency, the following expression,

$$\frac{P_{min}}{M_d} = \frac{2RT}{n} (C_F - C_d) \left[ \frac{C_b}{C_b - C_F} \ln \frac{C_b}{C_F} + \frac{C_d}{C_F - C_d} \ln \frac{C_d}{C_F} \right]$$
(32)

where  $P_{min}/M_d$  is the minimum power required to desalt a unit mass of water from  $C_F$  to  $C_d$ .

Equation (32) is compared with the approximate analysis for the cell pair potential<sup>[5]</sup> which can be expressed nondimensionally, as follows:

$$V = \beta \psi I - [(1 + \beta) \ln (1 - I) + (1 + t_a \beta / t_k) \ln (1 - t_k I / t_a)] + 2 \ln C \quad (33)$$

Where

$$V = E_c F/RT$$
  

$$\beta = F^2 D / \Lambda t_a RT$$
  

$$i = I / I_{max}$$
  

$$I_{max} = FDC_d / t_a \delta$$
  

$$C = C_b / C_d$$
  

$$\psi = (b - 2\delta) / \delta$$

First term in the RHS of equation (33) represents the resistive losses, second term is the polarization losses and the third term accounts for the useful work done against the membrane potentials. Using the model shown in Fig. 3 and assume one dimensional flow, a mass conservation for the dilute stream yields the following expression,

$$dy = (FbU_d / I) dC_d$$
  
= (b U\_d t\_a \delta / DC\_d) dC\_d (34)

Equations (33) and (34) give the changes of the flow and power in relation to the feed and product conditions. Consider the case of NaCl feed solution to the ED unit, such changes are shown in Fig. 4 to 7. Here, the product water salinity is fixed at 500 ppm and the equivalent conductance of the solution is equal to 109 cm<sup>2</sup>/ohm.gm.equ., while the boundary layer thickness is assumed constant and equals to 0.05 mm.



FIG. 3. Schematic ED unit for minimum power consumption derivation.

## Discussion

Figure 4 shows the polarization ratio changes along the stack length for a compartment thickness of 1 mm with flow velocities ranging from 10 to 30 cm/s. The gradient of the polarization ratio along the flow passage is reduced by increasing the flow velocity. However, higher flow velocities are associated with longer stacks.





FIG. 4. Effect of dilute velocity on polarization ratio.

The variations in the concentration ratio along the stack length for different flow velocities are shown in Fig. 5. At a fixed position from the entrance section, the concentration ratio is reduced by increasing the flow velocity.



FIG. 5. Effect of dilute velocity on concentration distribution.

The effect of changing the compartment thickness on the concentration ratio along the flow channel is as shown in Fig. 6. With a flow velocity of 10 cm/s, increasing the compartment thickness will increase the stack length and the concentration ratio gradient is reduced by increasing the compartment thickness.



FIG. 6. Effect of compartment thickness on concentration distribution.

Figure 7 demonstrates the influence of changing the compartment thickness on the polarization ratio along the flow direction. Here, the thinner the compartment the steeper the polarization distribution.

A comparison for the power required to produce one unit mass of the product is shown in Fig. 8. It also demonstrates the effect of changing the compartment thickness on the power required based on equations (33) & (34) using a feed salinity of 2000-10000 ppm NaCl. The figure shows that power increases as the compartment thickness increases. Besides, the minimum to the approximate power ratio varies from 62-46 percent at  $C_f$  of 2000 ppm to 26-19 percent at  $C_f$  of 10000 ppm by using a



FIG. 7. Effect of Compartment Thickness on Polarization Ratio.

compartment thickness of 0.5-1 mm, respectively. It should be noticed that the power calculated using the approximate analysis is about 20 percent of the reported actual power consumption<sup>[1,6]</sup>.

However, equations (33) & (34) do not account for practical losses such as electrode losses, concentrate compartment losses, ohmic losses in the membrane, ... etc., which may account for the difference between practical and approximate analysis.

### Conclusion

The present article presents an easy to follow procedure for analysing ED systems. It also provides a simple method of estimate, by hand calculations, the minimum



FIG. 8. Energy costs comparison.

energy required to operate an electrodialysis unit. The ratio between the minimum power required and that based on the approximate analysis ranges from 62-46 percent for feed of 2000 ppm to 26-19 percent for feed of 10000 ppm by varying the compartment thickness 0.5-1 mm for each case, respectively.

# Nomenclature

a activity

A membrane area

- *b* compartment thickness
- C concentration
- Di diffusion coefficient
- E potential drop
- F Faraday's contant
- *i* polarization ratio
- *I* current intensity
- J flux
- *K* electric conductance
- *l* membrane thickness
- $M_d$  desalted output
- *n* number of cells
- *p* power required
- R gas constant
- *t* transport number
- T absolute temperature
- U flow velocity
- *u* ion mobility
- V dimensionless potential drop
- X ion exchange capacity
- *x* coordinate across cell thickness
- *y* coordinate along stack length
- Z ion valence

## Subscripts

- a anions
- b brine
- c cell pair
- d desalted
- *df* diffusion
- *e* electrical
- f feed
- k cations
- *m* mean
- *min* minimum

## **Greek letters**

- $\phi$  electrical potential
- $\mu$  chemical potential
- $\gamma$  activity coefficient
- *A* equivalent conductance
- $\delta$  boundary layer thickness
- $\lambda$  ratio of mean activity coefficient

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يقدم هذا البحث تحليلا حسابيا لعملية الإعذاب باستخدام الديلزة الكهربائية ، ولقد استُنبطت المعادلات التفاضلية للتدفق الأيوني وكثافة التيار وفاقد الجهد لوحدة ديلزة كهربائية بدلالة كل من التركيز الأيوني ومعاملات الانتشار الأيوني في كل من الماء والأغشية ، وأعداد الانتقال الأيوني الكهربائية وخواص الأغشية . ولقد تم حل هذه المعادلات للحصول على فاقد الجهد لوحدة الديلزة بالإضافة إلى التدفق الأيوني لكل من الأيونات المساعدة والمعاكسة خلال الأغشية ، ومن ثم تقدير القدرة المطلوبة لتشغيل وحدة الديلزة بضرب فاقد الجده في التيار الكهربائي بعد تكامله على طول مسار التدفق . ولقد تمت مقارنة القدرة الحدية الدنيا المحسوبة بنتائج التحليل التقريبي للديلزة تحت ظروف تشغيل مختلفة .

أوضحت النتائج أن النسبة بين القدرة الحدية الدنيا والقدرة التقريبية تتراوح بين (٦٦ - ٦٦) بالمائة عند نسبة تركيز ٢٠٠٠ جزء بالمليون و (٢٦ - ١٩) بالمائة عند نسبة تركيز ١٠٠٠٠ جزء بالمليون ، بينا كان عرض حجيرة التدفق بين الأغشية يتراوح بين (٥, ٠ - ٠,١) مم على الترتيب .

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