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Digital Linear Filtering Method to Transform Schlumberger to Wenner Electric Resistivity Data

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Abstract. An approach for transforming Schlumberger apparent electric resistivity data into the corresponding Wenner apparent electric resistivity data is developed based on the digital linear filter theory. The auxiliary resistivity functions are used to determine the resistivity filters. The filter coefficients necessary for the transformation are derived, using the Schlumberger auxiliary resistivity function as input, and the Wenner auxiliary resistivity function as output. The Fourier Transform concept is applied to determine the frequency spectra of both the input Schlumberger, and the output Wenner functions. Finally, the filter coefficients are obtained by applying the inverse Fourier transformation after multiplying the frequency spectrum of the filter function by the frequency spectrum of the sinc function. The amplitude and phase spectra of both the Schlumberger and Wenner auxiliary resistivity functions are computed, and then the amplitude and phase spectra of the filter function could be calculated.

Apparent resistivity curves of some Schlumberger soundings, sampled in a rate of six points per logarithmic decade, are prepared and convolved with the calculated transformation filter coefficients to, obtain the corresponding Wenner sounding curves. The output transformed Wenner apparent resistivity curves are essentially the shifts of the Schlumberger curves, along the abscissa axis. The amount of this shift is found comparable with that given in literature.

Introduction

The present paper is an extension of Das and Kumar (1979) work in which digital linear filters were developed to transform Wenner to Schlumberger ones. Here, the filters are designed to perform the reverse procedure (*i.e.*, transform Schlumberger to Wenner).

In comparing the Wenner and Schlumberger arrays, it is noted that the electric current electrode spacing predominantly controls the depth of penetration in both configurations. For equal values of current electrode spacing, the depth of penetration reached in the two electrode configurations is, thus nearly equal. Generally, Schlumberger array has a greater probing depth and more resolving power than Wenner array. Also, because the potential electrodes remain in fixed location in the Schlumberger array, the effects of near surface variations in resistivity are reduced. In Wenner array, because all electrodes are moved for each reading, this array type would be more susceptible to near-surface lateral variations in resistivity. Furthermore, the voltage difference between the potential electrodes is considerably larger in the Wenner array than in the Schlumberger array, for the distance between the potential electrodes is larger in the Wenner configuration. For comparable depth of penetration, then, the reading accuracy of the voltage is better in the Wenner configuration than in the Schlumberger configuration (Koefoed, 1979 and Ward, 1990).

Measurements with the Schlumberger configuration, on the other hand, provide the interpreter with a unique advantage over other configurations; *i.e.*, the ability to identify and quantitatively and systematically correct for lateral, near-surface resistivity variations which might otherwise be interpreted in terms of vertical resistivity layering. Despite these differences, however, engineers are more familiar with the Wenner configuration, especially in studying near-surface resistivity distributions for determining the electrical earthing for constructing electrical facilities. This is basically due to the fact that, resistivity survey used for electric earthing is only for the shallow section of the earth's soil, where the engineered structures would be constructed. Therefore, the designed coded resistivity survey to control corrosion of buried structures, such as oil-refinery structures, boiler structures, and petrochemical structures is that with the Wenner electrode configuration (ASTM, 2001).

Since most of the resistivity surveys are conventionally carried out using the Schlumberger electrode array, because it is the most widely used array and hence possesses the most developed interpretation technique, it is, therefore, of practical importance to provide the engineers, geophysicists, and geologists with practical means, to obtain detailed apparent resistivity data of the very near section of the soil, without repeating the survey with the Wenner array. This means that considerable field advantages are derived by combining Schlumberger and Wenner field procedures in one single survey.

Kunetz (1966) was the first to indicate the possibility of applying the method of digital linear filtering in resistivity interpretation. Das and Kumar (1979) introduced the transformation of Wenner to Schlumberger apparent resistivity data, based on the digital linear filter theory. Ghosh (1971a; and 1971b) worked out the method of digital linear filtering in an improved form. He declared that the applicability of the method arises from the fact that the relations between the resistivity transform and the apparent resistivity functions are linear relations. Under this condition, the apparent resistivity functions are also linearly related. In the method of digital linear filtering, the values of one of the functions are obtained as a linear expression in the sample values of the other function (Parasnis, 1997). The samples are being taken at a constant interval along the abscissa axis. The coefficients in this linear expression are called the filter coefficients. Based on the same theory, we can present a filter to perform the reverse operation; namely, the transformation of Schlumberger apparent resistivity measurements into the corresponding Wenner apparent resistivities. The review of theoretical background draws heavily on Koefoed (1979). Morsy (1998) discussed, in detail, the Fourier transformation concept and the regime involved in the transformation from time domain to frequency domain and vice versa. He also reviewed the basic mathematics of convolution and deconvolution algorithms and the derivations of auxiliary resistivity functions of Wenner, Schlumberger, and dipole-dipole configurations (Koefoed, 1979).

Transformation of Configurations

Several authors have attempted the transformation of the apparent resistivity curves of one configuration into the other using the digital linear filter theory. Kumar and Das (1977) transformed dipole-dipole to Schlumberger sounding curves by means of a digital linear filter. Filters for this transformation are presented for the radial, perpendicular, and parallel (30º) dipole-dipole configuration. A sampling interval of $(1/6)ln10$ was used to obtain an accurate filter. The operation is handy and fast. The filter function of the dipole-Schlumberger transformation is determined in the usual way from the input dipole and output Schlumberger auxiliary resistivity functions. This is done by obtaining the filter amplitude spectrum for dipole-Schlumberger transformation, as the ratio of amplitude spectrum for their Kernel transformations. The filter phase spectrum, is the difference between the phase spectrum of the Schlumberger-Kernel, and that of the dipole-Kernel. The filter sinc response can then be obtained by inverse Fourier transformation.

Kumar and Das (1978) have also transformed Schlumberger apparent resistivities into dipole apparent resistivities over layered earth by the application of digital linear filters. The filter for this transformation was also designed using a sample interval of *(1/6)ln10* to provide accuracy, as well as fast performance. The amplitude and phase spectra of the Schlumberger-dipole transformation filter were determined such that the former, is the inverse of the amplitude spectrum of the dipole-Schlumberger transformation filter of Kumar and Das (1977), and the latter, is the negative of the phase spectrum of the dipole-Schlumberger transformation filter. Inverse Fourier Transformation then obtained the filter sinc response.

Deppermann (1954) developed a scheme to transform Wenner apparent resistivity curves into Schlumberger curves. Koefoed (1968), based on Deppermann's work, arrived at an explicit relationship between the two apparent resistivities as follow:

$$\rho_{aW} = 2a^2 \int_a^a \frac{\rho_{aS}}{s^2}\, ds \ . \tag{1}$$

Where: ρ_{aW} = Wenner apparent resistivity

ρ_{aS} = Schlumberger apparent resistivity

a = electrode spacing in the Wenner configuration.

s = half current electrode spacing in Schlumberger configuration.

When the average value of ρ_{aS}/s^2 is approximately linear in the interval $s = a$ to $s = 2a$, following Koefoed, one can write Eq. (1) as

$$\frac{\rho_{aS}}{s^2} = \frac{\rho_{aW}}{2a^2} \tag{2}$$

From Eq. (2) it was noted that ρ_{aW} is approximately equal to ρ_{aS} at "*s*" equal to "$a\sqrt{2}$". This suggests that the Schlumberger collection of the standard graphs may be used for curve matching the Wenner data if one puts the cross at $s = 1.4$ and $\rho_a = 1$. Geophysicists used this popular approximate method to interpret the Wenner data until Orellena and Mooney supplemented their original 1966 collection with a Wenner set.

Das and Kumar (1979) designed their Wenner-Schlumberger Transformation filter following the same procedures suggested by Ghosh (1971a) and the mathematical expressions for the input Wenner and the output Schlumberger auxiliary resistivity functions applied the Fourier transformation.

Schlumberger-Wenner Transformation

In the present paper, the filter coefficients for the transformation of Schlumberger apparent resistivity curves into the Wenner curves is derived, utilizing the Schlumberger auxiliary resistivity function as input, and the Wenner auxiliary resistivity function as output using the auxiliary (partial) resistivity functions (Koefoed, 1979). Knowing one pair of the input (Schlumberger auxiliary function) and the corresponding output (Wenner auxiliary function), both functions are sampled at a rate of six points per logarithmic decade (a sampling interval (Δx) of $\ln 10/6 = 0.383764$) to ensure accuracy.

The procedure is then first to determine the frequency spectra of the input Schlumberger and the output Wenner functions by the application of the Fourier Transform concept. The frequency spectrum of the filter function is then obtained by dividing these two frequency spectra; the filter function itself can be finally obtained by applying inverse Fourier transformation to its spectrum. According to the theory of digital linear filtering, the filter coefficients, which we wish to derive, are not the sample values of the filter function, but those of a function called the sinc response of the filter (Ghosh, 1971a and Das and Kumar, 1979). The sinc response of the filter is the convolution of the filter function with a sinc function of a period equal to twice the sampling interval (*e.g.* Koefoed, 1979). To obtain the filter coefficients, the filter function must be convolved with the sinc function or, more conveniently, the frequency spectrum of the filter function must be multiplied by the frequency spectrum of the sinc function.

According to the theory of Fourier analysis, the frequency spectrum of a sinc function is a rectangular function, *i.e.*, is a function, which is zero for frequencies higher than the cut off frequency. This result corresponds with the assumption implied by the sampling theory that the frequency spectrum of the function contains no frequencies higher than the reciprocal of twice the sampling interval. Thus, an intermediate step is required in the procedure for determining the filter coefficients. After the frequency spectrum of the filter function has been determined, this spectrum is cut off at the critical frequency (Nyquist Frequency) of "$1/(2\Delta x)$" and multiplied by a constant factor "Δx". The frequency spectrum of the sinc response of the filter is thus obtained. The filter coefficients are then obtained by applying inverse Fourier transformation.

The Filter Performance

The filter function for the Schlumberger-Wenner transformation is determined from the input Schlumberger and the output Wenner auxiliary resistivity functions. The amplitude and phase spectra of the Wenner and Schlumberger auxiliary resistivity functions are shown in Fig. (1 and 2), respectively.

The amplitude spectrum of the filter function is computed by dividing the amplitude spectra of the Wenner and Schlumberger functions whereas the phase spectrum of the filter function is computed by subtracting the phase spectra of the Wenner and Schlumberger functions. The computed filter spectra are both shown in Fig. (3).

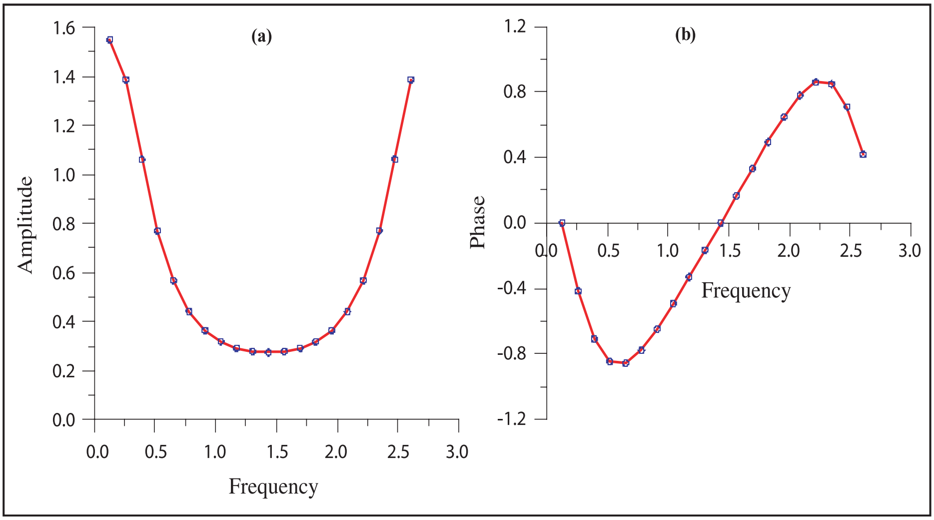

Fig. 1. (a) Amplitude, and (b) phase spectrum of the Wenner auxiliary resistivity function.

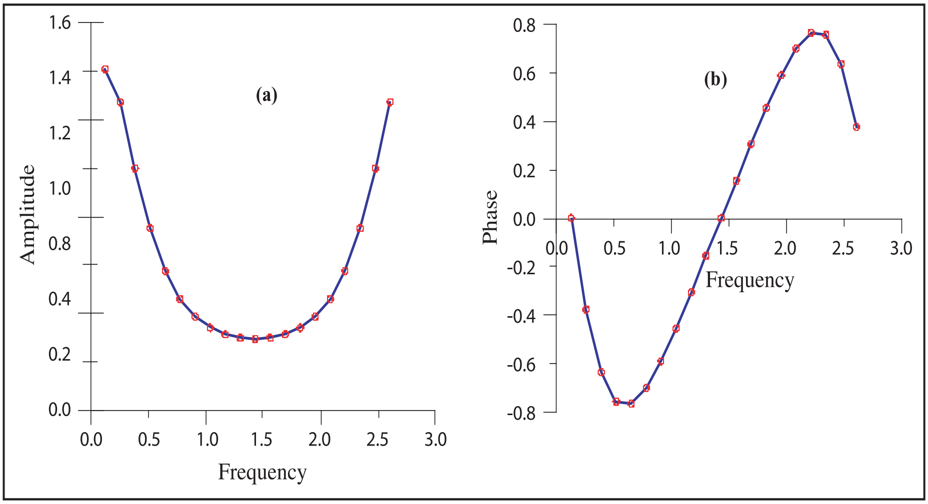

Fig. 2. (a) Amplitude, and (b) phase spectrum of the Schlumberger auxiliary resistivity function.

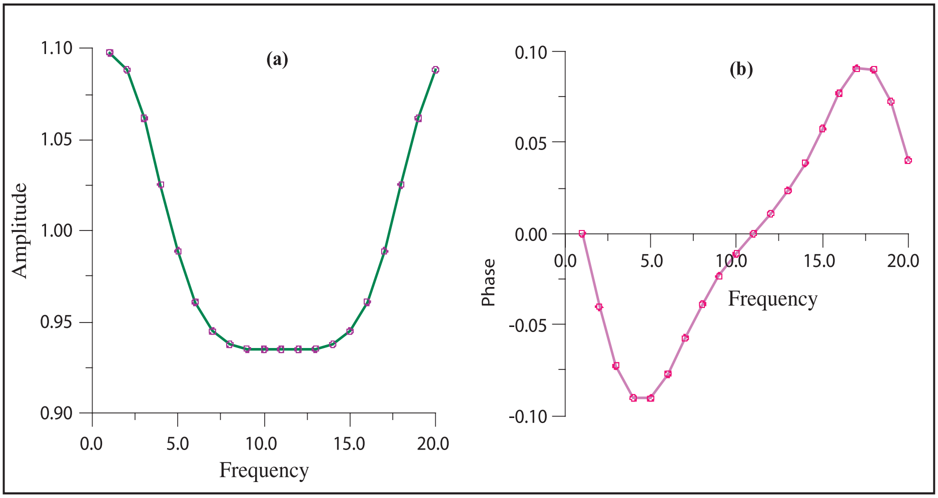

Fig. 3. (a) Amplitude, and (b) phase spectra of the filter function.

The Digital Filter Coefficients

The sinc response of the filter is reconstructed at an interval of *(1/6) ln10* and is shown in Fig. (4). The transformation is both accurate and fast. The designed Schlumberger-to-Wenner transformation filter is further checked against that designed by Das and Kumar (1979) for the transformation of Wenner to Schlumberger apparent resistivities, which in fact represent its inverse filter.

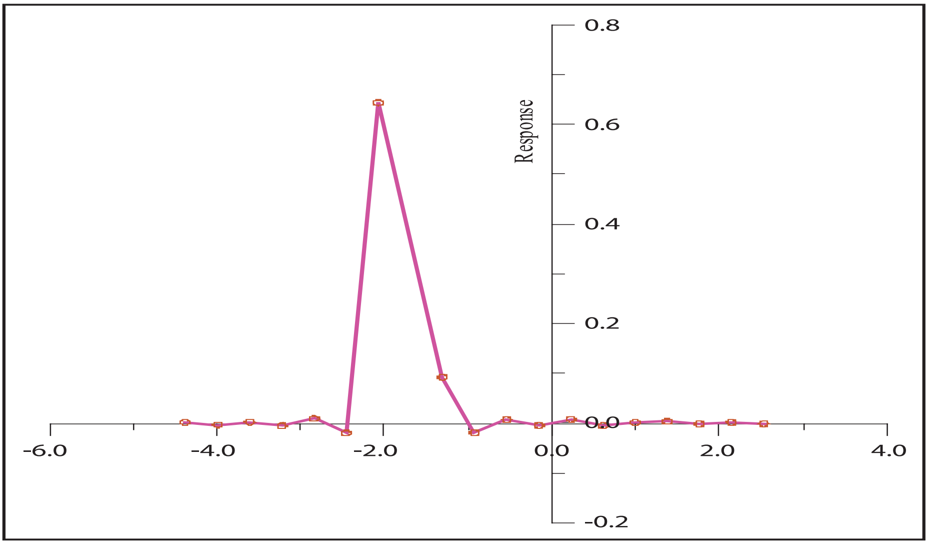

Fig. 4. Sinc response of the Schlumberger-Wenner filter function.

Table 1 shows the comparison between both the computed Schlumberger-to-Wenner filter coefficients and that of Wenner-to-Schlumberger given by Das and Kumar (1979).

Table 1. Comparison between computed Schlumberger-to-Wenner and Wenner-to-Schlumberger (Das and Kumar, 1979) transformation filter coefficients.

Abscissa	Schlumberger-to-Wenner transformation filter coefficients (Computed)	Wenner-to-Schlumberger transformation filter coefficients (Das and Kumar, 1979)
–4.3749	0.0015	0.0156
–3.9912	–0.0033	–0.0386
–3.6074	0.0018	0.0477
–3.2236	–0.0053	–0.0594
–2.8399	0.0090	0.0752
–2.4561	–0.0201	–0.0974
–2.0723	0.2926	0.1295
–1.6886	0.6432	–0.1744
–1.3048	0.0922	0.2515
–0.9210	–0.0194	–0.3724
–0.5373	0.0071	0.5834
–0.1535	–0.0046	–0.9896
0.2303	0.0076	1.9171
0.6140	–0.0050	–0.4094
0.9978	0.0009	0.1915
1.3815	0.0033	–0.1135
1.7653	–0.0015	0.0786
2.1491	0.0014	–0.0539
2.5328	–0.0015	0.0185

Conceiving that the Wenner-Schlumberger transformation filter spectrum $F_{WS}(f)$ has an amplitude spectrum $A_{WS}(f)$ and phase spectrum $\varphi_{WS}(f)$ and our Schlumberger-Wenner transformation filter $F_{SW}(f)$ has an amplitude $A_{SW}(f)$ and phase spectrum $\varphi_{SW}(f)$, if both filters were each other inverses, then $A_{SW}(f)$ should be the reciprocal of $A_{WS}(f)$, and $\varphi_{SW}(f)$ should be identical and of opposite sign to $\varphi_{WS}(f)$. Therefore, the multiplication of the two filter spectra, $F_{WS}(f)$ and $F_{SW}(f)$, must give a constant amplitude spectrum with zero phase. Alternatively, the convolution of the two filter sinc responses should yield a spike at zero lag. This can be shown mathematically as:

$$F_{ws}(f) = / A_{ws}(f) / \, e^{i\partial ws\,(f)}$$

$$F_{sw}(f) = / A_{sw}(f) / \, e^{i\partial sw\,(f)}$$

If $F_{WS}(f)$ is the inverse of $F_{SW}(f)$, then:

$$/A_{ws}(f)/ = 1/ / A_{sw}(f) / \text{ , and}$$

$$\partial_{ws}(f) = - \partial_{sw}(f);$$

That is, $F_{ws}(f) = 1 / / A_{sw}(f) / \, e^{-i\,\partial sw\,(f)}$, and

$$F_{ws}(f) \, . \, F_{sw}(f) = 1$$

Alternatively, $F_{ws}(x) * F_{sw}(x) = spike$

This means that, the convolution of the filter coefficients in Table (1) can be shown to be a spike.

Application

The computed filter coefficients given in Table (1) can now be used to transform measured Schlumberger resistivity sounding data (measured ρ_S) into their corresponding Wenner data (calculated ρ_W) by convolution according to the equation:

$$\rho_w(x) = \sum_{j=1}^{n} C_{j\rho_s}(y_i) \, , \tag{3}$$

Where $\rho_W(x)$ is the output Wenner apparent resistivity data, C_j is the filter coefficients, and $\rho_S(y_i)$ is the input Schlumberger apparent resistivity data. Then, we can confirm this transformation by measuring corresponding Wenner data to be compared with the computed one (calculated ρ_W).

Four Schlumberger resistivity soundings were selected from a resistivity survey consists of 25 soundings performed at the 15th May City, Southeast Cairo, Egypt (El-Behiry *et al.*, 1999) at sampling rate of six points per logarithmic decade (Fig. 5). The selected Schlumberger sounding data are extrapolated backward and forward, for the filter to fully apply to the first and last point of the input data, as the length of the filter is nineteen coefficients. These data are then convoluted with the calculated transformation filter coefficients given in Table (1) to obtain the corresponding Wenner sounding curves.

The unique condition for applying this filter is that, the Schlumberger resistivity data preferred to be measured with a sampling rate as six points per decade. If the field Schlumberger data were measured using a sampling rate more or less than of six points per decade, then these data can be plotted, digitized and extrapolated to obtain the data in the regime of six points per decade.

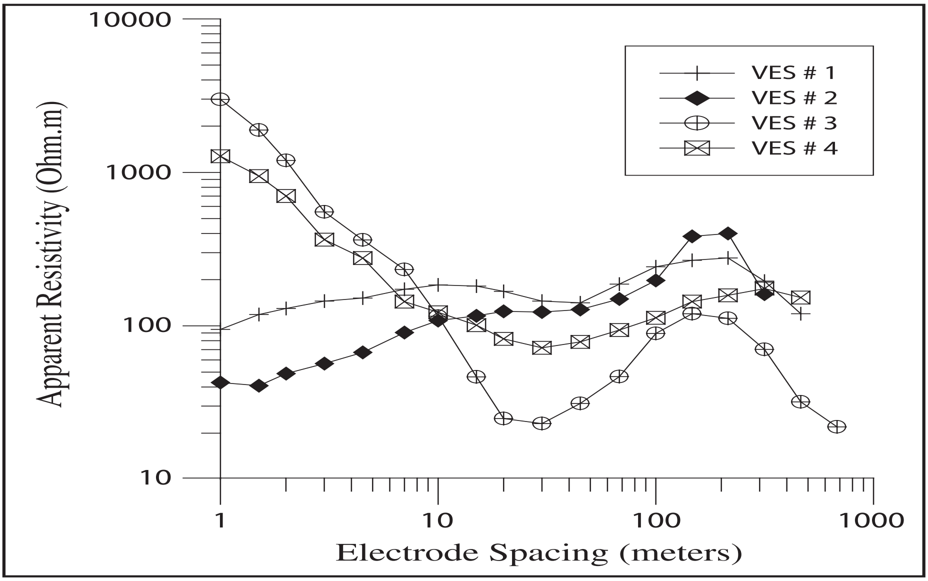

Fig. 5. Selected Schlumberger apparent resistivity sounding curves (after El-Behiry *et al.*, 1999).

The output transformed Wenner apparent resistivity curves and the input Schlumberger apparent resistivity curves of the selected sounding curves are shown in Fig. (6). The graphs in Fig. (6) readily demonstrate that, according to Eq. (2), the filter essentially shifts the Schlumberger curves along the abscissa axis to yield the Wenner curves. The amount of shifts along the abscissa axis (1.42m) is in agreement with the relationship between Wenner and Schlumberger apparent resistivities given by Telford *et al*., (1990, see equations 8.42a and 8.42b, respectively), which equals 1.39m.

Figure 7 shows the RMS error-relation between the interpreted Schlumberger models and the transformed Wenner models of 15 soundings, which were carried out at the 15th May City, Southeast Cairo, Egypt (El-Behiry *et al*., 1999). The resulted correlation coefficient of 0.93 indicates the validity of the developed transformation procedure in obtaining plausible resistivity model for the shallow as well as the deep sections of the investigated soils.

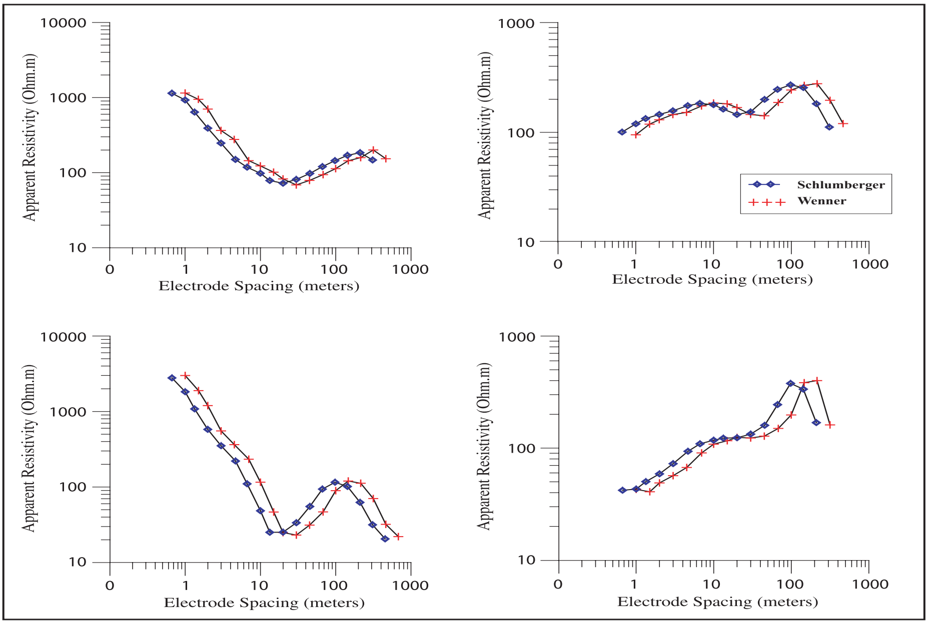

Fig. 6. Transformed Schlumberger-to-Wenner soundings apparent resistivity curves.

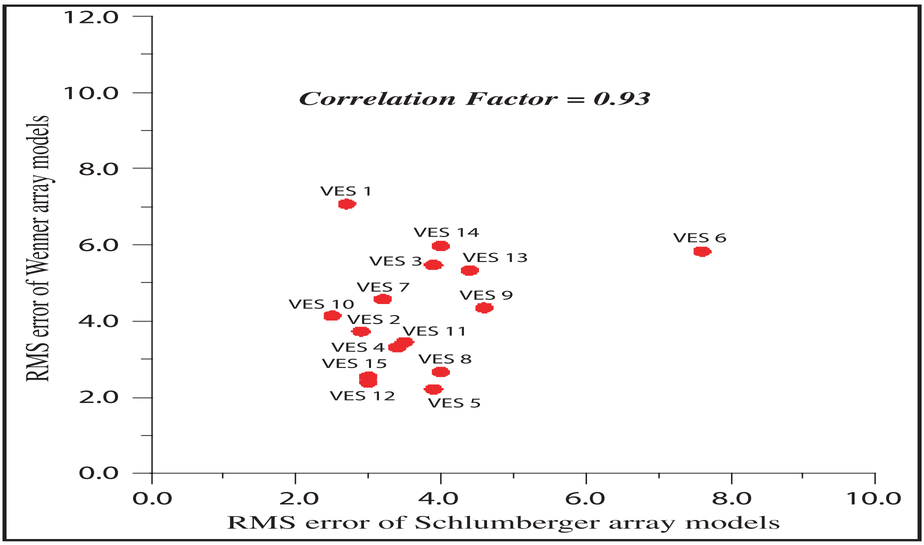

Fig. 7. Comparison between RMS errors for Wenner and Schlumberger models.

Conclusion

An approach for transforming Schlumberger apparent electric resistivity data is developed based on the digital linear filter theory. The coefficients of the resistivity filter necessary for the transformation are derived using the auxiliary resistivity functions of both arrays. The Fourier Transform concept is applied to determine the frequency spectra of both arrays functions. Finally, the filter coefficients are obtained by applying the inverse Fourier transformation after multiplying the frequency spectrum of the filter function by the frequency spectrum of the sinc function. The amplitude and phase spectra of both the Schlumberger and Wenner auxiliary resistivity functions are computed and then the amplitude and phase spectra of the filter function could be calculated.

The developed filter coefficients are tested using apparent resistivity curves of some Schlumberger soundings, sampled in a rate of six points per logarithmic decade. The apparent resistivity values are convolved with the calculated transformation filter coefficients to obtain the corresponding Wenner sounding curves.

Comparing the Schlumberger with the transformed Wenner curves for the same resistivity distribution reveals that, the filter essentially shifts the Schlumberger curves along the abscissa axis to yield the Wenner curves. The amount of shifts along the abscissa axis is found in agreement with that given by Telford *et al.* (1990).

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طريقة الفلترة الخطية الرقمية لتحويل بيانات المقاومة الكهربائية الأرضية المقاسة بتشكيل شلمبرجر إلى تشكيل وينر

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المستخلص. تم تطبيق تقنية الفلترة الخطية الرقمية لتحويل بيانات المقاومة الكهربائية الأرضية، والمقاسة بتشكيل شلمبرجر، والمستخدم بكثرة في الدراسات الهيدروجيوفيزيائية، إلى بيانات للمقاومة الكهربائية الأرضية، مقاسة بتشكيل وينر، والمستخدم في الأغراض الهندسية.

تم استنتاج معاملات الفلتر الرقمي المستخدم في التحويل، من دالات المقاومة الكهربائية الأرضية لكلا التشكيلين، وذلك بتطبيق طريقة تحويل فوريير لتعيين طيف التردد لدوال كلا التشكيلين. أما ثوابت الفلتر فقد تم حسابها بتطبيق مقلوب فوريير، من حاصل ضرب طيف تردد دالة الفلتر بطيف تردد دالة سينك. ومن ثم تم حساب السعة وطيف الطور لدالات المقاومة الكهربية الإضافية، لكل من التشكيلين، ومنها تم حساب السعة، وطيف الطور للفلتر الرقمي المستخدم في التحويل.

تم اختبار فلتر التحويل الرقمي، وذلك بتطبيقه على عدد من بيانات المقاومة الكهربائية الأرضية، والمقاسة بتشكيل شلمبرجر، بغرض تحويلها إلى بيانات للمقاومة الكهربائية الأرضية، مقاسة بتشكيل وينر. تم ملاحظة أن بيانات المقاومة الكهربائية الأرضية المحولة لتشكيل وينر تعطي نفس بيانات المقاومة، ولكن على مسافات أقل للأقطاب الكهربائية وهذه النتيجة وجدت متوافقة إلى حد كبير لما هو مستنتج في المراجع العلمية المتخصصة في دراسة بيانات المقاومة الكهربائية الأرضية، المقاسة بأكثر من تشكيل.