TSS - An Engineering Software for Analysis of Thin Sections under Stress

A. Abou Ezz, H.A. Bogis, M. Akyurt and A.A.A. Alghamdi

Mechanical Engineering Department, King Abdulaziz University
P.O. Box 9027, Jeddah 21413, Saudi Arabia

e-mail: a_ez@email.com, hbogis@hotmail.com, makyurt@kaau.edu.sa, aljinaidi@hotmail.com

ABSTRACT

TSS, an interactive engineering software in Visual Basic, is presented for the analysis of thin sections under the influence of applied loads. *TSS* enables the user to readily generate any cross-sectional shape desired, and to modify it. The software then computes geometric properties of the generated section. These include the area, centroidal coordinates, first and second moments of inertia, and the shear center.

TSS additionally computes the stress distribution in elastic thin sections of arbitrary shapes under various types of loading. Using *TSS*, a shear flow plot along the limbs of the thin section can be displayed, and the twist angle of the section as well as the maximum shear flow value and its location can be determined. The values of the shear flow at the start and end nodes of each limb can be tabulated. Principal normal and shear stresses are readily computed at any location on the section, and their direction cosines are reported. The octahedral shear stress is also displayed.

With the aid of *TSS,* open channels can be analyzed as well as combinations of up to ten closed cells. The Mohr circle is used as a convenient graphical method for the presentation of computed stresses.

KEYWORDS

geometric property, shear center, shear flow, software, stress analysis, thin section.

1. INTRODUCTION

Structural sections that are fabricated from thin plates or by extrusion have become increasingly important due to their economy of material use and ease of manufacture. Most structural members used in land, sea and aircraft are basically an assembly of stiffened shell structures ranging from single-celled closed sections like the body and the fuselage to multicellular assemblies consisting of combinations of open and closed cells such as those used in wings and tail surfaces.

Calculation of stresses and failure modes in thin-walled structural members is a complex procedure. Structural designers often need assistance in analyzing such structures. Computer programs can be used as suitable vehicles in this regard. The latter trend becomes quite obvious when going through recent literature involving thin-walled cylinders. Thus Alfano and associates [1] outlined a procedure for the automatic determination of the shear center, and evaluation of the overall state of stress in multi-cell thin walled sections subject to axial force, bending moment, shearing force as well as torque. They used graph theory for this purpose, and applied their method to study a ship's hull. A computer model for the analysis of systems with semi-rigid connections on 3-D thin walled framed structures was proposed by Conci [2] and Papangelis [3]. The procedure enables the study of cross-sectional properties, warping displacements, and longitudinal and shear stresses for thin walled open and closed cross-sections of arbitrary shapes. The study also considers buckling.

Ueda and Masaoka [4] noted that thin walled structures such as ships and offshore structures are composed of many stiffened plate panels. To analyze these structures, the authors proposed a plate element using eigen-functions for large deflections and sectional yield conditions. In a related study, Grant [5] used network theory for determining the shear center of asymmetric sections. Pollock and co-workers [6] carried out investigations to determine shear centers for anisotropic elastic thin-walled composite beams, cantilevered and loaded transversely at the free end. They discovered that twisting may occur for composite beams even if shear forces are applied at the shear center. Mentrasti [7] studied the shear-torsion state of stress in a curved beam with a cross-section of a thin rectangle with sides not parallel to the plane of the beam.

A number of authors studied the torsion of thin-walled structures. Thus Grant [5] studied the pure torsion (bending-free) of prismatic bars. He noted that the axis of twist must be located at the shear center of the section. To determine the shear center, the author isolated the contribution of warping. In a related study the same author [8] applied network theory to the Saint-Venant torsion in thin-walled multi-cell sections by treating the section as a network of interconnected limbs. The author observed that network methods offer advantages since they require far fewer equations than the total number of problem variables. Still other researchers [9 and 10] formulated finite elements for the torsion of thin-walled beams, including warping in torsional loading. Studying the shear-torsion state in a curved beam, Mentrasti [7] evaluated the shear-torsion moment of inertia in multi-connected thin-walled cross sections.

Papangelis and Hancock [3] studied the elastic buckling of thin-walled open and closed cross sections in structural members. They developed a computer program whereby the buckle half-wave length of the member can be varied, producing the buckled shape. Ohga and associates [11], on the other hand, considered the buckling of closed cross sections only. They utilized the method of transfer matrix in their analytical study, and reported that local and overall elastic buckling loads for thin-walled members can be obtained. They also illustrated a technique for the estimation of buckling mode shapes. Djugash and Kalyanaraman [12] presented a numerical method for the nonlinear and instability analysis of thin-walled members that are subjected to biaxial bending.

Hasham and Rasmussen [13] reported on the results of a series of compression and major-axis bending tests performed on three different lengths of thin-walled I-sections fabricated from high strength steel plates by welding. Chick and Rasmussen [14, 15] also conducted tests to assess the behavior and design of thin-walled I-sections in combined compression and minor-axis bending.

In what follows *TSS*, an interactive and user friendly software, is introduced for computing geometric properties and the stress distribution in elastic thin sections of arbitrary shapes under the influence of suitably specified loads. The sections are assumed to comprise an arbitrary number of limbs of uniform thickness, attached to each other at end nodes, and that the wall thickness is sufficiently small compared to other dimensions of the section.

2. INERTIA AND STRESS PROPERTIES

A number of relationships were used during the current study, and numerous assumptions were made. Below is a brief listing of some of these.

The centroidal moments of inertia I_{xx}, I_{yy} and I_{xy} are computed relative to a Cartesian system of coordinates located at the centroid of the section in question. The angle □ of the principal moments of inertia of this section as well as their magnitudes may be computed by

$$\tan 2\alpha = \frac{2I_{xy}}{I_{yy} - I_{xx}} \qquad [1]$$

$$I_{\max} = \frac{I_{xx} + I_{yy}}{2} + 0.5\sqrt{(I_{xx} + I_{yy})^2 + I^2_{xy}} \qquad [2]$$

$$I_{\min} = \frac{I_{xx} + I_{yy}}{2} - 0.5\sqrt{(I_{xx} + I_{yy})^2 + I^2_{xy}} \qquad [3]$$

For a member that is subjected to bending moments M_x and M_y, the normal stress distribution on its cross section is given by

$$\sigma_{zz} = -\left(\frac{M_y I_{xx} + M_x I_{xy}}{I_{xx} I_{yy} - I_{xy}^{\ 2}}\right)x + \left(\frac{M_x I_{yy} + M_y I_{xy}}{I_{xx} I_{yy} - I_{xy}^{\ 2}}\right)y \qquad [4]$$

In asymmetrical bending, the orientation of the neutral axis of the section can be expressed by

$$\tan\alpha = \left(\frac{M_x I_{xy} + M_y I_{xx}}{\mathrm{M}_x I_{yy} + \mathrm{M}_y I_{xy}}\right) \qquad [5]$$

where □ is positive when measured in the tawafwise (counter-clockwise) direction.

For thin sections comprising a number of limbs, it may be assumed that the limbs are joined to each other at their ends (nodes) only, and that the thickness of a given limb is uniform. When it is further assumed that the limb thicknesses are small when compared to limb lengths, the total shear flow q within a limb may be expressed by the following expression, where q_0 is the initial shear flow in the limb, q_1 and q_2 are constants, and s is the distance along the limb.

$$q = q_0 + q_1 s + q_2 s^2 \qquad [6]$$

A final relationship on shear flow may be cited as [16]

$$\oint \frac{q}{Gt}\, ds = 2A\theta \qquad [7]$$

where G is the modulus of rigidity of the material
A is the area enclosed by the cell, and
□ is the twist of the compartment.

3. THE *TSS* SOFTWARE

The program is initiated by clicking on its icon on the desktop, upon which the *Main Menu* (Fig. 1) with its logo appears. Under the *File* menu are listed the tool items *New, Open, Save, Save as,* and *Exit*. Figure 2 shows the *Input Data* form which appears when data is imported by invoking

File / New or *File / Open*, and then selecting a data file. Using the latter form, the user describes the thin section by filling a *limb table,* where each limb consists of a thickness *t*, and a pair of *end nodes*, and a *node table,* in which each node is characterized by its *x* and *y* coordinates. The information contained in the *limb table* and *node table* in Fig. 2 is for the section shown in Fig. 3. The *File / Open* option retrieves an existing data file which contains the geometric data for the given section, the applied load set, and the material properties for the section.

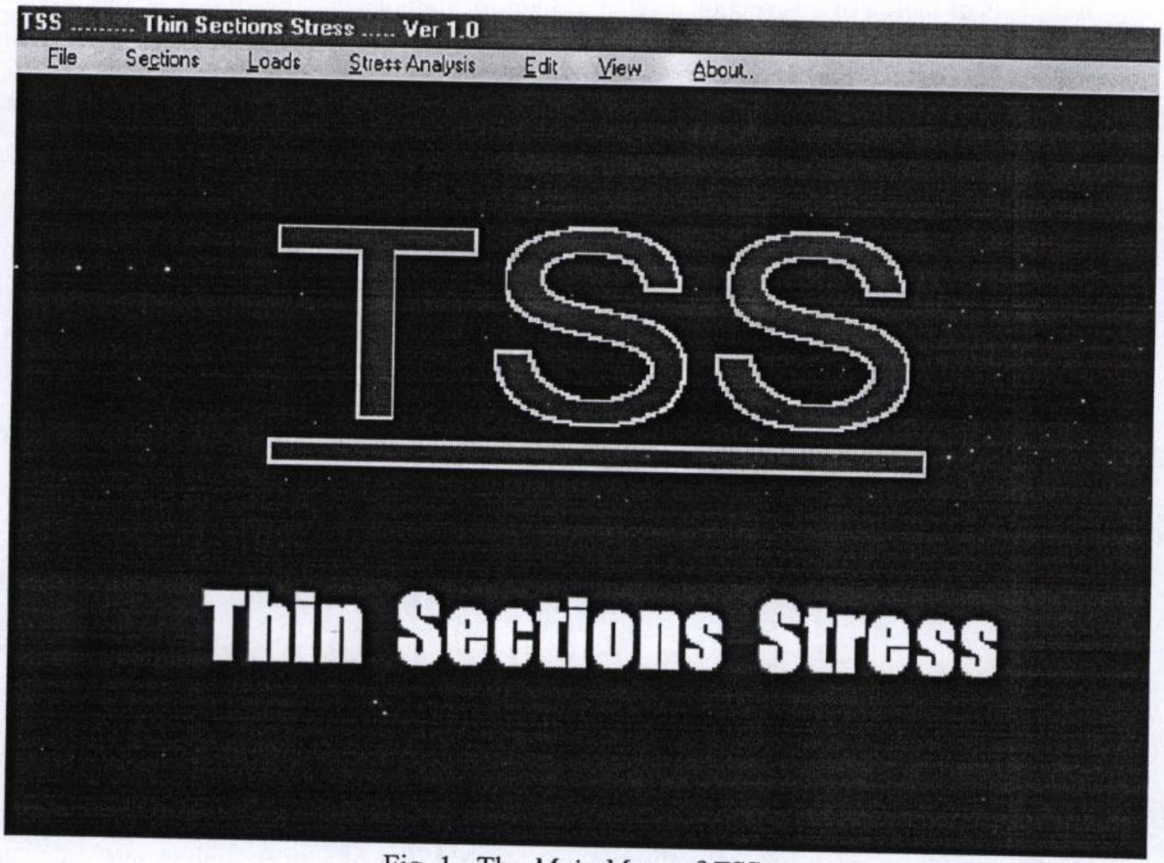
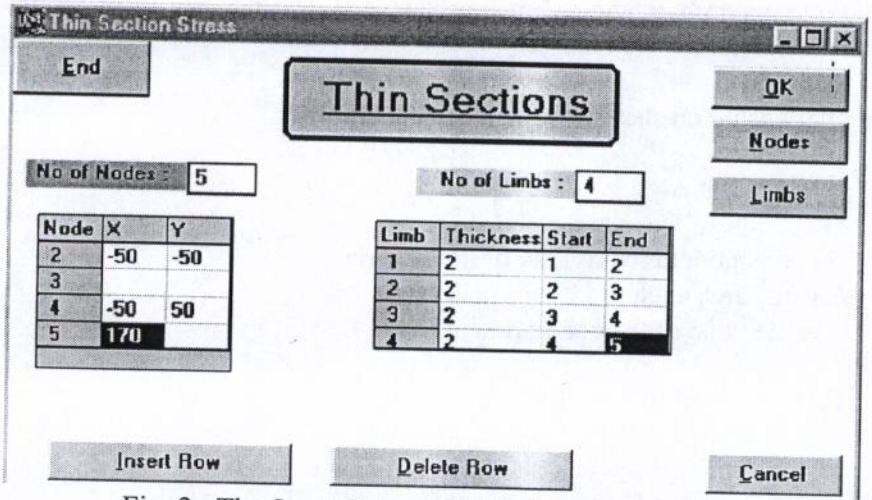


Fig. 1. The *Main Menu* of *TSS.*



No of Nodes : 5

No of Limbs : 4

Node	X	Y
2	-50	-50
3		
4	-50	50
5	170	

Limb	Thickness	Start	End
1	2	1	2
2	2	2	3
3	2	3	4
4	2	4	5

Fig. 2. The *Input Data* and *Thin Sections* window.

The *Input Data* form further lists the *Number of Nodes*, the *Number of Limbs*, the *Insert Row* and *Delete Row* buttons as well as an *Arc Add* button. The *End* and the *OK* buttons close the *Input Data Form* and return the activity to the *Main Menu.* Invoking *Cancel* causes the discarding of any changes made in the present menu.

Invoking *Sections* on the *Main Menu* opens the *Thin Sections* menu of Fig. 2. This menu enables the user to portray the cross-section in question by specifying the *number* of its *nodes*, and the number of its *limbs* by filling appropriate text boxes. Considering the section shown in Fig. 3, it is to be observed that the section has five limbs (walls), and the meeting points of the limbs, called nodes , are denoted as □ , □ ,□ and □ . For the coordinate system shown, the coordinates of nodes 1 and 4 are given as [120, 0] and [0, 50], respectively. The remaining nodes are symmetrically located. The thickness of limbs 1 to 4 are 2 mm, and the thickness of limb 5 is 5 mm.

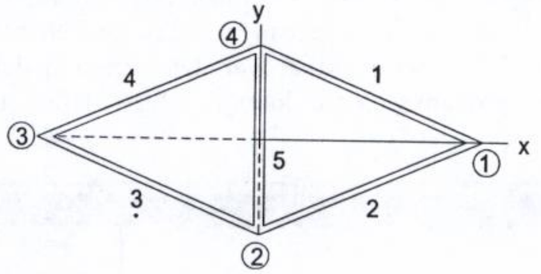


Fig. 3. Illustration of nodes and limbs.

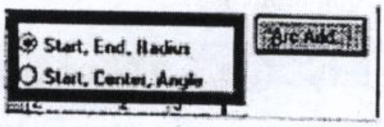


Fig. 4a. Use of the *Arc Add* button.

As soon as the user specifies the number of nodes and number of limbs in the *Thin Sections* menu, two tables (Fig. 2) are generated by *TSS*, where relevant data for the nodes and limbs can be entered. Thus the user describes the geometry of the thin section by the use of the *Limb Table,* with each limb consisting of a thickness *t*, a pair of *End Nodes*, and the *Node Table,* in which each node is represented by its *x, y* coordinates. Nodal coordinates are established by utilizing a global coordinate system with its origin located at any convenient point.

Located at the right edge of the *Thin Sections Form* is the *Arc Add* button. When invoked, this button enables the user to generate sections that are formed of circular arcs alone or circular sections that are connected to straight limbs. The user has two options in defining the required arc, as shown in Fig. 4a. Opting for the first option (*Start, End, Radius*) produces the *Arc Addition* menu (Fig. 4b) to define the start and end nodes for the arc and its radius.

The *Arc Add* menu of Fig. 4a further enables the user to specify the connectivity conditions of nodes. To this end four options are available for identifying the connectivity of the start and end nodes of the arc. In each case the user can enter the required data for specifying the two end nodes.

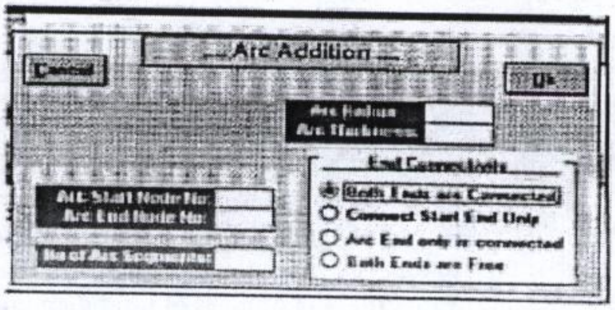


Fig. 4b. The menu for *Arc Addition.*

When the lower option in the *Arc Add* menu of Fig. 4a is selected, the arc must be defined by its start node, center and the arc angle. In both cases the user defines the number of arc segments, where the default number of arc segments is 8. When specifying the start and end nodes of an arc, care must be exercised such that the arc is generated in the tawafwise (counter clockwise) direction, i.e., from the start node to the end node.

Invoking the *Cancel* button in the *Arc Addition* menu enables the user to return to the *Thin Section* menu without adding an arc. If, on the other hand, the *OK* button, which is located on the top right corner of both *Arc Addition* menus, is clicked on, this action signals the acceptance of the data to *TSS*. *TSS* then generates the corresponding mesh of limbs with the specified thicknesses, and defines the starting and end node for each limb. In fact clicking on the *OK* button on one of the *Arc Addition* menus causes the activation of a series of *TSS* routines to solve the problem. To this end *TSS* displays the *Work Sheet* menu (Fig. 5), and proceeds to ensure the connectivity of the limbs. The software automatically identifies whether the section is open or closed. When a closed or multicellular section is discovered, the program determines the independent circuits from among the probable closed loop(s) which define the cells or compartment(s).

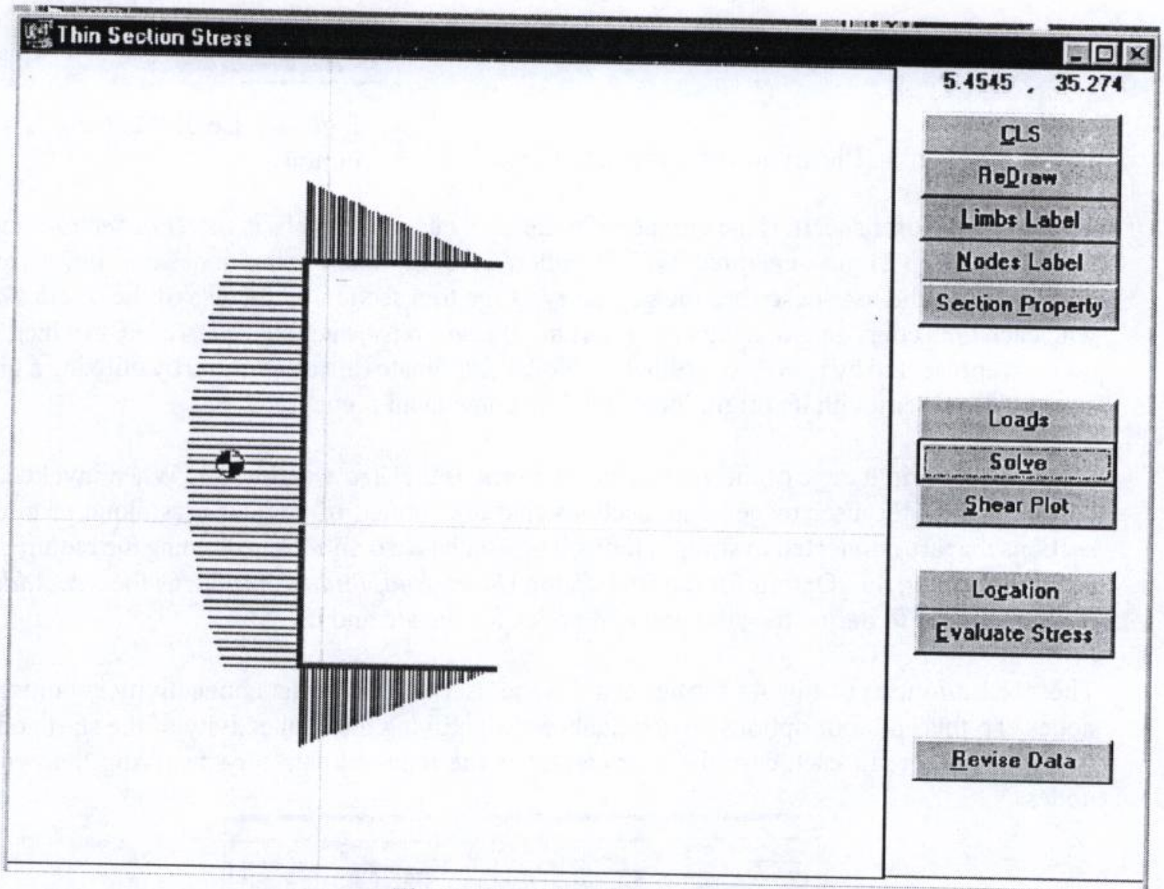


Fig. 5. The *Work Sheet* menu.

TSS further demarcates the limb elements belonging to the identified compartment, and utilizes this information in calculating the area enclosed by each closed path and for the determination of shear flow. The study of stresses in torsion members and beams may require the determination of geometrical properties of the section. These may include its area, the coordinates of its centroid and its shear center, moments of inertia, the product of inertia and the polar moment of inertia. *TSS* expresses these values for coordinate axes located at the centroid of the section, although the user may use a separate origin of his own. The orientation of the principal axes of inertia that is reported by *TSS* is determined in the tawafwise (counterclockwise) direction from the user-defined x-axis.

At this stage the user is in a position to display properties of the section (Fig. 8), and to plot the shear flow for the pre-defined set of loads. Likewise the user can specify the location at which the stress state needs to be determined. Once such a location is specified, *TSS* proceeds to display the principal normal stresses and the shear stress values at that point, together with the corresponding Mohr circle representation as well as the direction cosines of the same.

Referring to the *Work Sheet* menu of Fig. 5, the two numbers displayed at the top right corner designate the coordinates of the cursor. All buttons in Fig. 5 can be actuated either by a left click of the mouse, or by pressing the *Alt* key while typing the underlined character. Invoking the *CLS* button causes the removal of the drawing displayed in the main screen area, whereas the *Re-Draw* button re-generates the cross section in the display. Clicking on the *Limbs Label* and *Nodes Label* buttons displays the numbers of the limbs and the labels of nodes, respectively, on the *Work Sheet*. Figure 6 depicts a U-section with node labels, and Fig. 6a shows a Z-section and a plot of the shear flow distribution along each limb. Figures 6b and 6c display distributions of shear flow in two closed sections.

Fig. 6. U-Section with node labels.

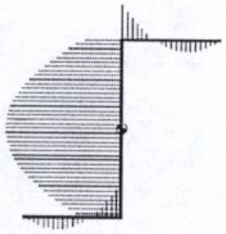


Fig. 6a. Shear flow in a Z-section.

Fig. 6b.

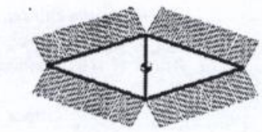


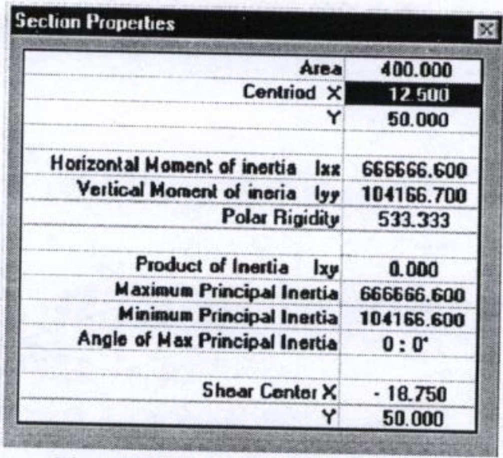
Fig. 6c.

Invoking the *Section Properties* button from among the column of buttons located on the right edge of the *Work Sheet* menu of Fig. 5 causes the displaying of the *Section Data* form (Fig. 7) that lists geometrical properties of the section. These include the coordinates of the centroid as well as the shear center relative to the user-defined coordinate frame. Additionally the cross sectional area, the second moment of area, the product of inertia, maximum and minimum principal values of inertia and the orientation of the former, and the polar rigidity of the section are displayed in the *Section Data* menu.

The *shear center* is defined as the position of that point on the section at which shear loads produce no twisting. The same point is also the *center of twist* of a section that is subjected to pure torsion. In practice a shear load does not generally act through the shear center. For analysis purposes, the shear load is replaced by an equivalent force-couple system comprising a shear force acting at the center and a couple. It is therefore necessary to determine the location of the shear center in all sections. *TSS* determines the shear center for closed sections as well as for open sections.

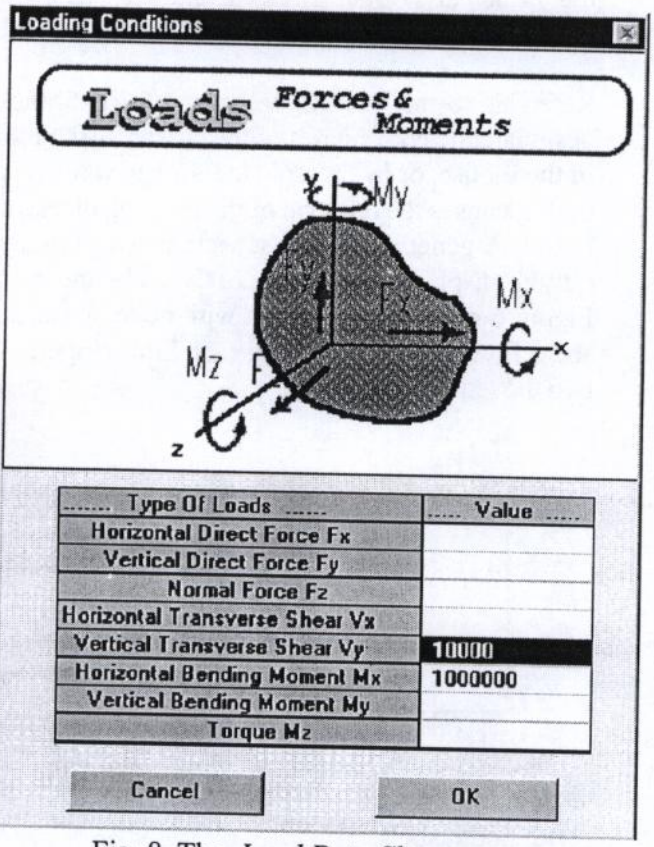
Clicking on the *Loads* button in the *Work Sheet* menu opens the *Load Data Sheet* of Fig. 8, where a set of loads acting on the cross section can be specified. The *Load Data Sheet* can be obtained also by invoking the *Loads* button from the *Main Menu.*

Referring to Fig. 8, the normal force Fz is the axial direct force normal to the section, while Vx and Vy are the transverse shear forces, and Mx, My and Mz are the bending and twisting moments in a direction defined according to the right hand rule. Forces and moments are referred to a Cartesian system of axes Oxyz, where x and y axes are in the plane of the section and the z axis is normal to the section at its shear center.



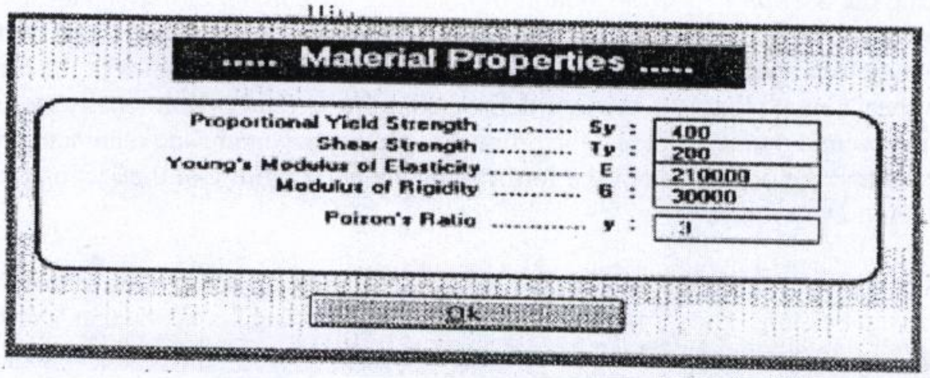
Area	400.000
Centriod X	12.500
Y	50.000
Horizontal Moment of inertia Ixx	666666.600
Vertical Moment of ineria Iyy	104166.700
Polar Rigidity	533.333
Product of Inertia Ixy	0.000
Maximum Principal Inertia	666666.600
Minimum Principal Inertia	104166.600
Angle of Max Principal Inertia	0 : 0°
Shear Center X	- 18.750
Y	50.000

Fig. 7. The *Section Data* form.



.... Type Of Loads	..... Value .....
Horizontal Direct Force Fx	
Vertical Direct Force Fy	
Normal Force Fz	
Horizontal Transverse Shear Vx	
Vertical Transverse Shear Vy	10000
Horizontal Bending Moment Mx	1000000
Vertical Bending Moment My	
Torque Mz	

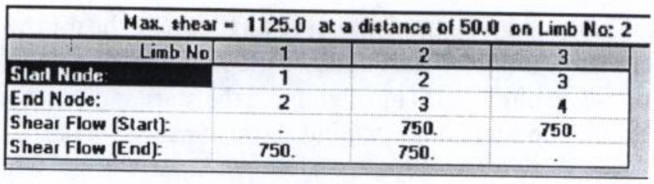
Fig. 8. The *Load Data Sheet* menu.



Proportional Yield Strength .... Sy : 400
Shear Strength .... Ty : 200
Young's Modulus of Elasticity .... E : 210000
Modulus of Rigidity .... G : 30000
Poison's Ratio .... v : .3

Fig. 9. The *Material Data* menu.

Invoking of the *Material* button on the *Work Sheet* menu opens the *Material Data Menu* of Fig. 9. Material properties that are entered via this menu are utilized by *TSS* in computing the polar rigidity of the section and its twist angle. Figure 9 displays the default set of values for material properties.



Max. shear = 1125.0 at a distance of 50.0 on Limb No: 2

Limb No	1	2	3
Start Node:	1	2	3
End Node:	2	3	4
Shear Flow (Start):	.	750.	750.
Shear Flow (End):	750.	750.	.

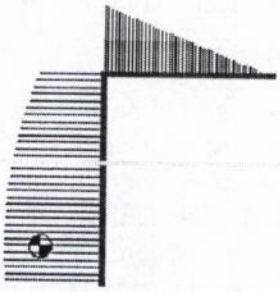


Fig. 10. The *Shear Flow Plot* menu.

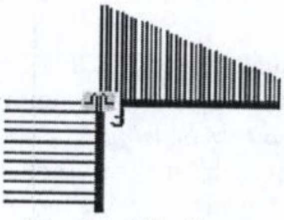


Fig. 11. The *Stress Sensing Icon.*

The *Solve* button on the *Work Sheet* menu instructs *TSS* to recalculate all according to the most recent modifications in load conditions and material properties, and displays the updated shear flow distribution. In similar manner, clicking on the *Shear Plot* button results in the displaying of shear forces along the section and the listing of a table containing the start and end node labels for each limb as well as the corresponding shear flow values. The *Shear Plot* menu (Fig. 10) further provides the user with the maximum shear flow value and the location of that point on the corresponding limb relative to the start node of the limb. It is to be noted that *TSS* can be utilized to compute the shear flow distribution in open and closed sections as well as in sections that are formed from a combination of open and closed components with multiple cells.

As a basic tenet, *TSS* assumes that thin sections comprise an arbitrary number of limbs, each of a uniform thickness, and attached to each other at end nodes, and that the wall thicknesses are small relative to other dimensions in the section. The shear stress is hence considered to be uniform across the wall thickness. For *purely shear loads* it follows then that shear flow varies quadratically along the limb.

The application of a *pure torque* to a beam with a closed section results in the development of a constant shear flow in the beam wall. The induced shear stress may vary, however, over the section if the wall thickness changes from limb to limb. For a thin-walled hollow torsion member with two or more compartments, shear flow values in each closed loop would be different. The shear flow value in such cases depends on the enclosed area of the cell, the length of the closed path, thickness of limbs comprising the loop, the modulus of rigidity of the material of the section, and the externally applied torque. The shear flow value in a limb neighboring two closed loops is the resultant of the shear flow in both loops.

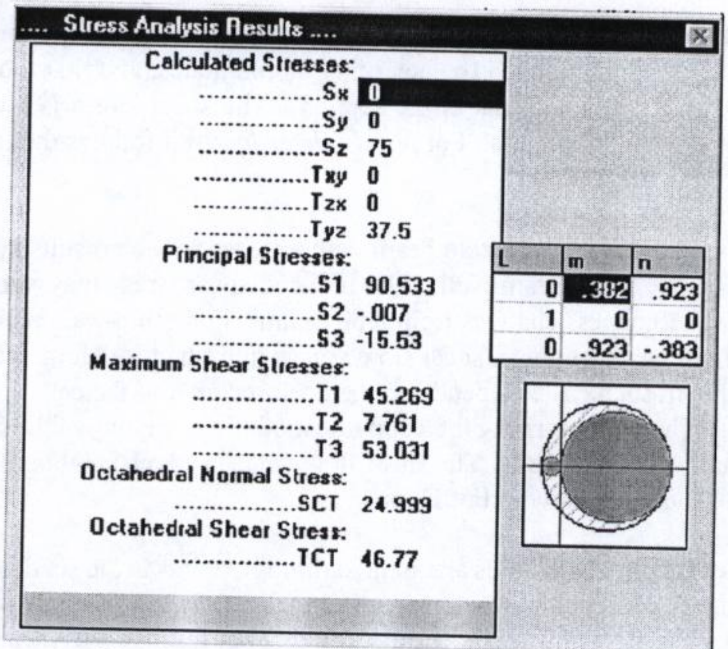
In most practical cases, shear loads are applied through points in the section that are not the shear center, meaning that in such cases *torsional as well as shear effects* are present. Shear loads in such circumstances are replaced by Vx and Vy that act through the shear center of the section together with a torque Mz. The user thus first defines the shear center, which is a property of the section. Next the external force is replaced by an equivalent system comprising the external force, which is now applied at the shear center, and a couple.

Two colors are used in the *shear-flow plot* of Fig. 10 to identify the direction of shear flow. Thus green represents shear flow in the direction from the start node of the limb towards its end, and red represents the shear flow in the opposite direction. Thus reversing the start and end nodes for a limb causes a change in the color of the plot, without affecting the shear flow values.

Due to the assumption that the thickness of each limb is uniform, and that the thicknesses of limbs are small compared to other dimensions, the *shear stress* at any point in the section is computed by dividing the shear flow value by limb thickness. The direction of shear stress is assumed to be parallel to the orientation of the limb.

Referring to the *Work Sheet* menu of Fig. 5, the *Location* button is used to supply *TSS* the coordinates of any point on the section where the state of stress is to be evaluated. To this end, a *Stress Sensing Icon* is utilized. Being anchored to the cross-hair of the mouse, this icon pops up each time the user activates the *Location* button. Left-clicking the mouse on the *Work Sheet* menu also releases the stress sensing icon at the current cursor coordinates. *TSS* ensures that the stress sensing icon floats to the nearest point on a limb (Fig. 11), and an enlarged view of the icon as perched on the limb appears at the upper right corner of the screen.

The *Evaluate Stress* button of Fig. 5, when actuated, instructs *TSS* to evaluate the stress state at the location of the *Stress Sensing Icon*. The value of direct stress at this location on the section is evaluated under the assumptions that a)plane sections of prismatic beams remain plane after displacement, and b)the material of the beam is homogeneous and linearly elastic. It is further assumed that any direct or transverse force applied on the section passes through its shear center. Any eccentric force is replaced by an equivalent force passing through the shear center and a couple. The value of stress at a point naturally depends on the location of the point, the applied loading and the geometric properties of the section. Stresses at a point due to different loads are then added by superposition to yield the elements of the stress tensor.



```
Calculated Stresses:
  ................Sx  0
  ................Sy  0
  ................Sz  75
  ................Txy 0
  ................Tzx 0
  ................Tyz 37.5
Principal Stresses:
  ................S1  90.533
  ................S2  -.007
  ................S3  -15.53
Maximum Shear Stresses:
  ................T1  45.269
  ................T2  7.761
  ................T3  53.031
Octahedral Normal Stress:
  ................SCT 24.999
Octahedral Shear Stress:
  ................TCT 46.77
```

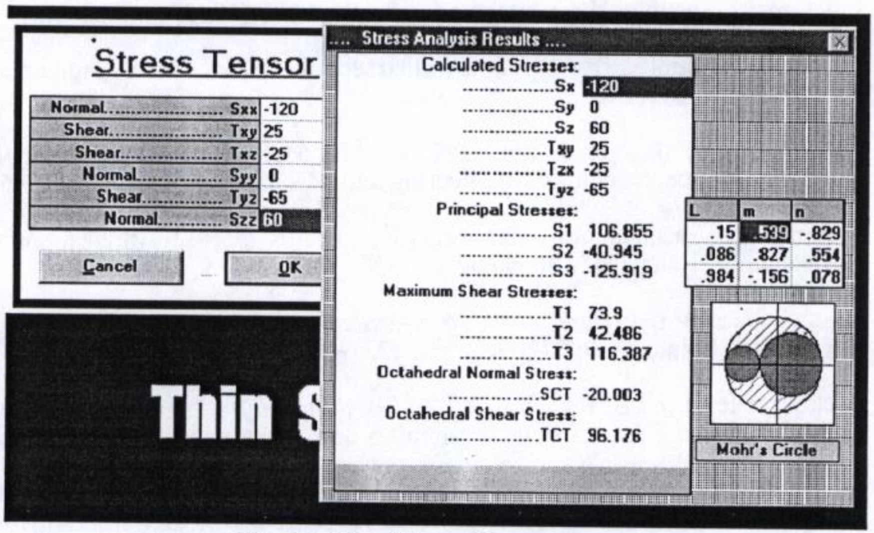
L	m	n
0	.382	.923
1	0	0
0	.923	-.383

Fig. 12. The *Stress Analysis Results* menu.

TSS displays the *Stress Analysis Results* menu of Fig. 12 when the *Evaluate Stress* button is invoked. For a point the coordinates of which have not been defined previously, *TSS* assigns the coordinates [0,0] for the current position. The menu lists all stresses at this point, including principal normal stresses, their direction cosines, maximum shear stresses, the octahedral normal stress as well as the octahedral shear stress. The Mohr circle, located at the lower right corner of the *Stress Analysis Results* menu, illustrates the relation between the shear and normal stresses acting on any plane through the selected location.

The *Hard Copy* button is used to print a copy of the figures located in the drawing area of the *Work Sheet* menu by using the default printer configured with Windows. Pressing on the *Revise Data* button signals the end of the stress analysis session, and reverts back to the *Thin Sections* menu. The user may then wish to modify geometric parameters by changing nodal and limbs data, or return further to the *Main Menu* for another session by invoking the *End* button at the top left of the *Thin Sections* menu.

It may be pointed out that one of the buttons that is located on the *Main Menu* is the *Loads* button. Activating this menu item enables the user to specify a set of loads to be applied on the section in spread-sheet fashion. The *Loads* button can be invoked also during the stress analyses session.



Normal......................	Sxx	-120
Shear......................	Txy	25
Shear..................	Txz	-25
Normal..............	Syy	0
Shear.............	Tyz	-65
Normal...........	Szz	60

Calculated Stresses:
..................Sx -120
..................Sy 0
..................Sz 60
................Txy 25
................Tzx -25
................Tyz -65
Principal Stresses:
..................S1 106.855
..................S2 -40.945
..................S3 -125.919
Maximum Shear Stresses:
..................T1 73.9
..................T2 42.486
..................T3 116.387
Octahedral Normal Stress:
.................SCT -20.003
Octahedral Shear Stress:
.................TCT 96.176

L	m	n
.15	.539	-.829
.086	.827	.554
.984	-.156	.078

Fig. 13. The *Stress Tensor* menu.

A yet another button to be found on the *Main Menu* is *Stress Analysis*, which is of general utility. The latter button helps users in processing stress tensors in general. Thus when the *Stress Analysis* button is invoked, *TSS* displays the *Stress Tensor* menu of Fig. 13, where the known elements of a tensor of interest can be manually entered. Pressing then the *OK* button commences the processing of the data. The results are displayed in the form of the *Stress Analysis Results* menu. Invoking *Cancel* leads back to the *Main Menu.*

4. CONCLUDING REMARKS

TSS is a powerful software for computing geometric properties of thin sections, and for determining the state of stress in elastic thin sections of arbitrary shapes under the influence of applied loads. To this end, *TSS* provides an easy means for the generation and modification of cross-sectional shapes. Geometric properties such as area, centroidal coordinates, first and second moments of inertia, and the shear center for specified sections are automatically determined. Plots of shear flow along the limbs of thin section are produced. Likewise tabulated values of shear flow at start and end nodes of each limb are detailed. The maximum shear flow value and its location are indicated. Principal normal and shear stresses and their direction cosines are reported at any desired location on the section. Octahedral shear stresses are computed. The Mohr circle is used as a convenient graphical method for the presentation of associated stresses. As such *TSS* seems to possess the potential to be useful for practicing engineers as well as engineering students.

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