J. Phys. B: At. Mol. Opt. Phys. 30 (1997) L777–L783. Printed in the UK PII: S0953-4075(97)85694-5

LETTER TO THE EDITOR

Squeezing-induced resonant finite refractive index—zero absorption

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Received 7 July 1997, in final form 1 September 1997

Abstract. We demonstrate that the spectra of a probe beam probing an ensemble of two-level atoms damped by a broadband squeezed vacuum and driven by a weak laser may exhibit a large refractive index accompanied by zero absorption when the probe beam frequency is tuned to the laser frequency. This resonant effect is due to coherent population oscillations. The same theoretical model also exhibits a large refractive index with zero absorption at the Rabi sidebands and near resonance. These results may be verified by present experiments.

Normally the refractive index of a gaseous medium near an optical resonance is high but this is also accompanied by high absorption. Recently, Scully [1] has shown that a three-level atom in the lambda configuration initially prepared in a phase-coherent state can exhibit a high refractive index accompanied by vanishing absorption. This unusual effect has also been reported for other schemes involving multilevel atoms [2]. Quang and Freedhoff have shown [3] that even a simple two-level system driven by a strong laser field can generate a large refractive index accompanied by zero absorption. This was further extended to include inhomogeneously broadened spectra [4]. An enhancement of the refractive index accompanied by vanishing absorption has recently been observed experimentally [5] in a coherently prepared Rb atom. This opens the possibility of many practical applications. It has been suggested [1] that optical materials having a large refractive index accompanied by vanishing absorption could have potential applications in fundamental and applied physics, for example, in high-precision magnetometry and optical microscopy.

In all of the models considered to date, a high refractive index accompanied by vanishing absorption is achieved away from the central frequency of the spectrum or a large refractive index with almost zero absorption occurs at the central frequency of the spectrum where the presence of a very strong laser is essential. Here, we propose that two-level atoms driven by a relatively weak coherent field and damped by a broadband squeezed vacuum can exhibit finite dispersion accompanied by zero absorption when the frequency of the probe beam is tuned to the laser frequency. This resonant effect thus offers the advantage of an optical medium exhibiting a large refractive index accompanied by zero absorption without saturating the atoms by strong coherent fields.

0953-4075/97/210777+07$19.50

The dispersion and absorption properties of a system of atoms can be investigated by using the Kramers–Kronig dispersion relations, which determine the complex susceptibility and hence the complex refractive index of a weak field probing the system. We consider two-level atoms driven coherently and damped by a broadband squeezed vacuum. The master equation for the reduced atomic density operator in a frame rotating at ω_L, i.e. the laser frequency is [6]

$$\begin{aligned}\dot{\rho} = -\mathrm{i}\Delta[S^z, \rho] + \mathrm{i}\Omega[S^+ + S^-, \rho] + \tfrac{1}{2}\gamma(N+1)(2S^-\rho S^+ - S^+S^-\rho - \rho S^+S^-)\\ +\tfrac{1}{2}\gamma N(2S^+\rho S^- - S^-S^+\rho - \rho S^-S^+)\\ -\gamma|M|\mathrm{e}^{-\mathrm{i}\Phi}S^+\rho S^+ - \gamma|M|\mathrm{e}^{\mathrm{i}\Phi}S^-\rho S^-\end{aligned} \tag{1}$$

where $\Delta = \omega_A - \omega_L$ and ω_A is the atomic transition frequency. $S^\pm$, S^z are the atomic operators and γ is the Einstein A coefficient. The Rabi frequency is 2Ω and the phase difference of the laser and squeezed vacuum is $\Phi = 2\phi_L - \phi$, where ϕ_L is the laser phase. The parameters N and $M = |M|\mathrm{e}^{\mathrm{i}\phi}$ describe squeezing such that $|M|^2 \leqslant N(N+1)$. The squeezed field is assumed to be centred about ω_A and the squeezed bandwidth is sufficiently broad so that the squeezed vacuum appears as a δ-correlated squeezed white noise to the atoms. Henceforth, we restrict ourselves to a squeezed vacuum with $|M|^2 = N(N+1)$.

We now assume that after the system (coherently driven atoms + squeezed vacuum) has attained equilibrium conditions, the atoms are perturbed by a weak probe field of frequency ω_P and amplitude E_P, the dispersion and absorption of which we are interested in. The linear susceptibility $\chi(\omega_P)$ of the probe beam at frequency ω_P is given in terms of the Fourier transform of the expectation value of the two-time commutator of the atomic operators [7]

$$\chi(\omega_P) = \mathrm{i}|\boldsymbol{\mu}\cdot\boldsymbol{E}_P|^2\tilde{N}\int_0^\infty \mathrm{d}\tau \lim_{t\to\infty}\langle[S^-(t+\tau), S^+(t)]\rangle \mathrm{e}^{\mathrm{i}\omega_P\tau} \tag{2}$$

where $\boldsymbol{\mu}$ is the transition electric dipole moment of the atom and $\tilde{N}$ is the atomic density. Equation (2) can be written as

$$\chi(\omega_P) = |\boldsymbol{\mu}\cdot\boldsymbol{E}_P|^2\tilde{N}(\chi'(\omega_P) + \mathrm{i}\chi''(\omega_P)) \tag{3}$$

where the real part $\chi'(\omega_P)$ determines the refractive index of the probe field, whereas the imaginary part $\chi''(\omega_P)$ determines the absorption coefficient. Using equations (1)–(3) we now illustrate the resonant finite refractive index–zero absorption effect involving relatively weak coherent driving fields.

At the central frequency ($\omega_P = \omega_L$) it can be shown that

$$\chi'(\omega_P = \omega_L) = \frac{2(\Delta/\gamma)\{n[(n/2)^2 + (\Delta/\gamma)^2 - |M|^2] + (2\Omega/\gamma)^2|M|\sin\Phi\}}{D^2} \tag{4}$$

$$\chi''(\omega_P = \omega_L) = \frac{2\{-2(\Omega/\gamma)^2(n|M|\cos\Phi + 2|M|^2) + \frac{1}{2}n^2[(n/2)^2 + (\Delta/\gamma)^2 - |M|^2]\}}{D^2} \tag{5}$$

where $D = \{n[(n/2)^2 + (\Delta/\gamma)^2 - |M|^2] + (2\Omega/\gamma)^2[\frac{1}{2}n + |M|\cos\Phi]\}$ and $n = 1 + 2N$. The probe beam is amplified when $\chi''(0) < 0$, and the medium is transparent when $\chi''(0) = 0$. The condition for amplification without population inversion at the central component of the absorption spectrum has been studied by Ficek *et al* [8].

It is obvious from equation (4) that the refractive index at the central frequency always vanishes when the system is on-resonance, i.e. $\Delta = 0$. Hence, finite detuning is responsible for the finite refractive index at the central frequency and it is found to increase with Δ. Consequently, from equations (4) and (5) when the probe beam frequency is tuned to the laser frequency, the necessary condition for a finite refractive index without absorption is given by

$$\left(\frac{\Delta}{\gamma}\right)^2 = \frac{8(\Omega/\gamma)^2[\frac{1}{2}n|M|\cos\Phi + |M|^2]}{n^2} - \frac{1}{4} \quad (>0). \tag{6}$$

In figure 1 we plot the absorption–dispersion spectrum for $N = 5$, $\Omega/\gamma = 0.3$, $\Phi = 0$ and $\Delta/\gamma = 0.3283$. The value of detuning is calculated from the condition (6) above. Clearly, there is a large refractive index accompanied by vanishing absorption at the central frequency. Furthermore, the line profile of the absorption spectrum and the refractive index are reversed; the usual absorption spectrum becomes dispersion-like, while the usual dispersion profile for the refractive index becomes absorption-like. This is called Rayleigh-wing formation (cf [4]). Presently available sources of squeezed light [9] generate a squeezed vacuum with $N \approx 0.5$. Hence, from the condition (6) we get $\Delta/\gamma \approx 0.2$ when $\Omega/\gamma = 0.3$, $N = 0.5$ and $\Phi = 0$; this makes the theoretical model accessible to the experiment. It is well known that the major experimental problem, in any attempt to observe squeezing-induced effects, is that present sources of squeezed light generate a beam which can couple only a small fraction of the modes enveloping the atom. However, it has been shown [10] that this problem can be avoided by placing the atom inside an optical cavity. The advantage of using a cavity is that the atom is coupled only to those modes which are contained in a small solid angle around the cavity axis [11].

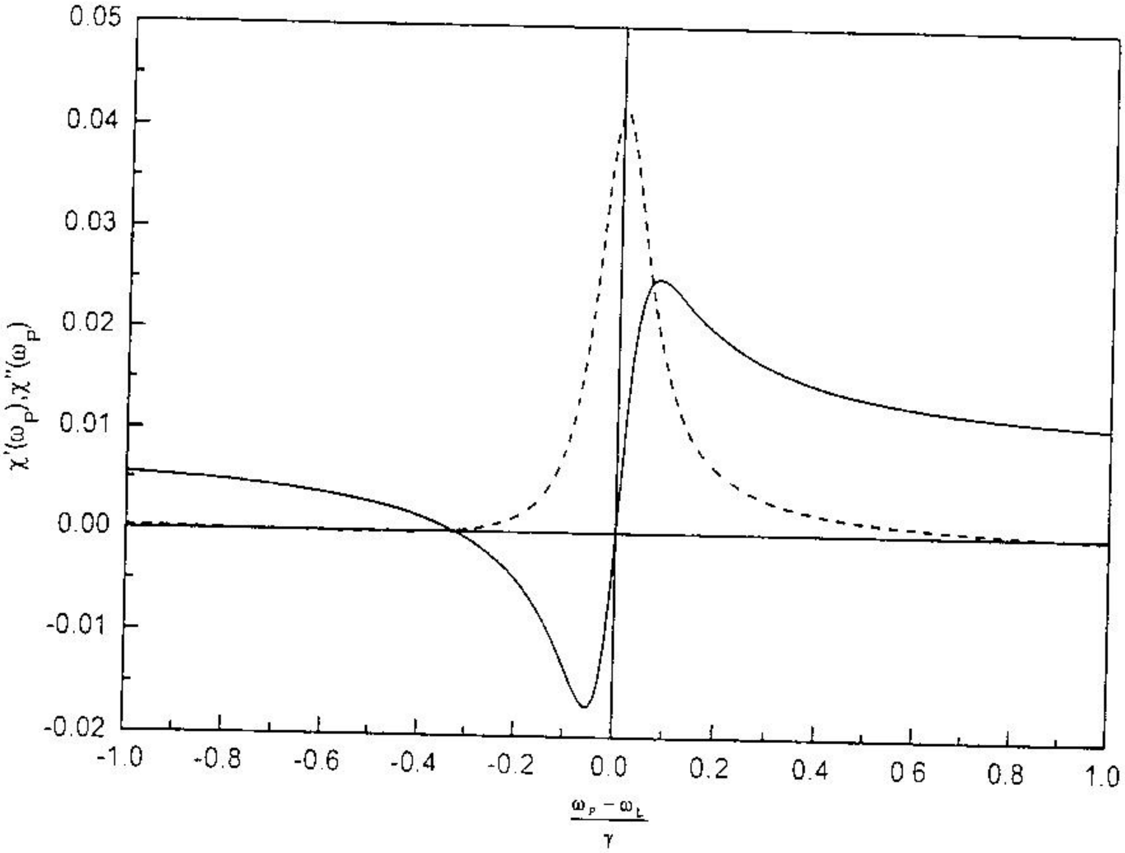

Figure 1. The real part $\chi'(\omega_P)$ (refractive index, broken curve) and the imaginary part $\chi''(\omega_P)$ (absorption spectrum, full curve) of the susceptibility of a probe beam probing two-level atoms damped by a broadband squeezed vacuum ($N = 5$, $\Phi = 0$) and driven by an off-resonant weak laser ($\Delta/\gamma = 0.3283$, $\Omega/\gamma = 0.3$).

We attribute the resonant finite refractive index–zero absorption effect to coherent population oscillations [8, 12]. These are induced by the driving and probe fields beating together at the difference frequency, $\delta = \omega_P - \omega_L$. They are responsible for the gain without population inversion at the central frequency of the absorption spectrum of a probe beam reported in [8]. The oscillations are significantly enhanced by the presence of the squeezed vacuum. In order to see this, it has been shown [8] that the steady-state dipole moment is given by

$$\langle S^- \rangle = \mathrm{i}\left[\frac{2(n - 2\mathrm{i}\Delta/\gamma)}{uH}\left(\frac{\Omega_P}{\gamma}\right) - \frac{4\Omega}{H\gamma}\left(n + 2|M|\mathrm{e}^{-\mathrm{i}\Phi} - 2\mathrm{i}\frac{\Delta}{\gamma}\right)\langle S^z \rangle_{(-\delta)}\right] \tag{7}$$

where $\langle S^z \rangle_{(-\delta)}$ is the amplitude of the coherent population oscillations induced by the probe beam, $H = n^2 + 4[(\Delta/\gamma)^2 - |M|^2]$ and $u = n + 8\Omega^2(n + 2|M|\cos\Phi)/H$. Clearly, the dipole moment is composed of two parts: the first part, proportional to Ω_P ($2\Omega_P$ is the Rabi frequency of the probe field), is driven directly by the probe field, while the second, proportional to Ω, results from the population oscillations. The second term, which is significantly enhanced by the squeezing, reduces the dipole moment of the atom. $\chi''(\omega_P)$ is proportional to the imaginary part of the dipole moment, and hence absorption can be reduced significantly by the coherent population oscillations. For the parameters of figure 1, it can easily be verified that the imaginary part of $\langle S^- \rangle$ vanishes. In figure 2 we illustrate how the coherent population oscillations affect the absorption spectrum with different detunings, but for the same parameters as in figure 1.

A finite refractive index without absorption can also be achieved near the central frequency ($|(\omega_P - \omega_L)/\gamma| \sim 0.5$). This is shown in figure 3 for a resonantly strong laser field

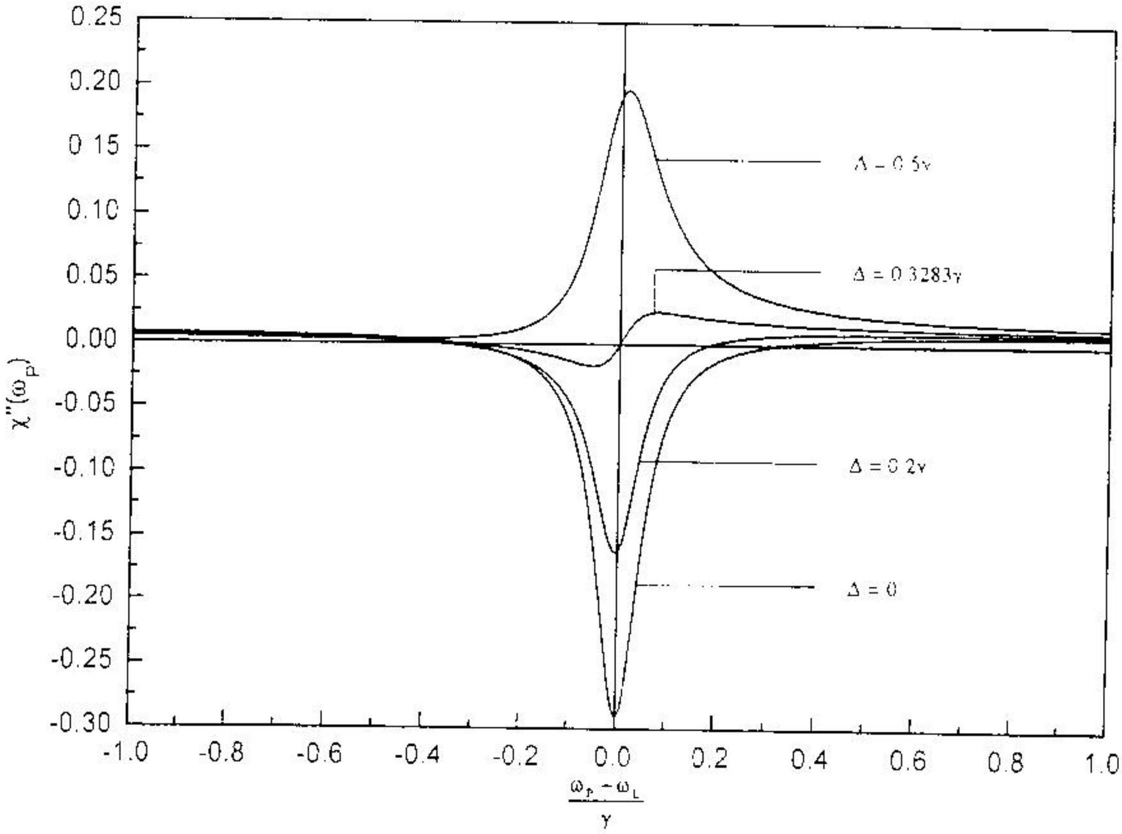

Figure 2. The absorption spectra corresponding to different detunings (Δ/γ), but with the same parameters as in figure 1.

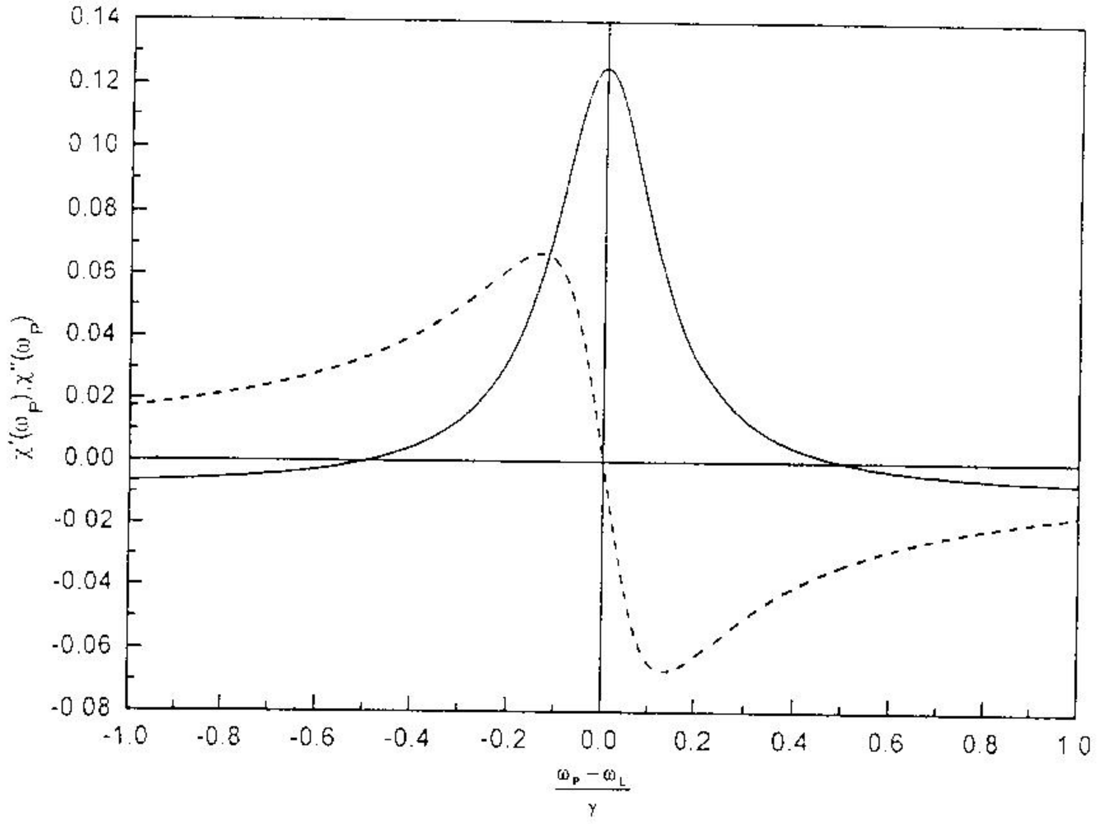

Figure 3. The same as in figure 1, but now with $N = 0.5$, $\Omega/\gamma = 5$, $\Delta/\gamma = 0$ and $\Phi = \pi$.

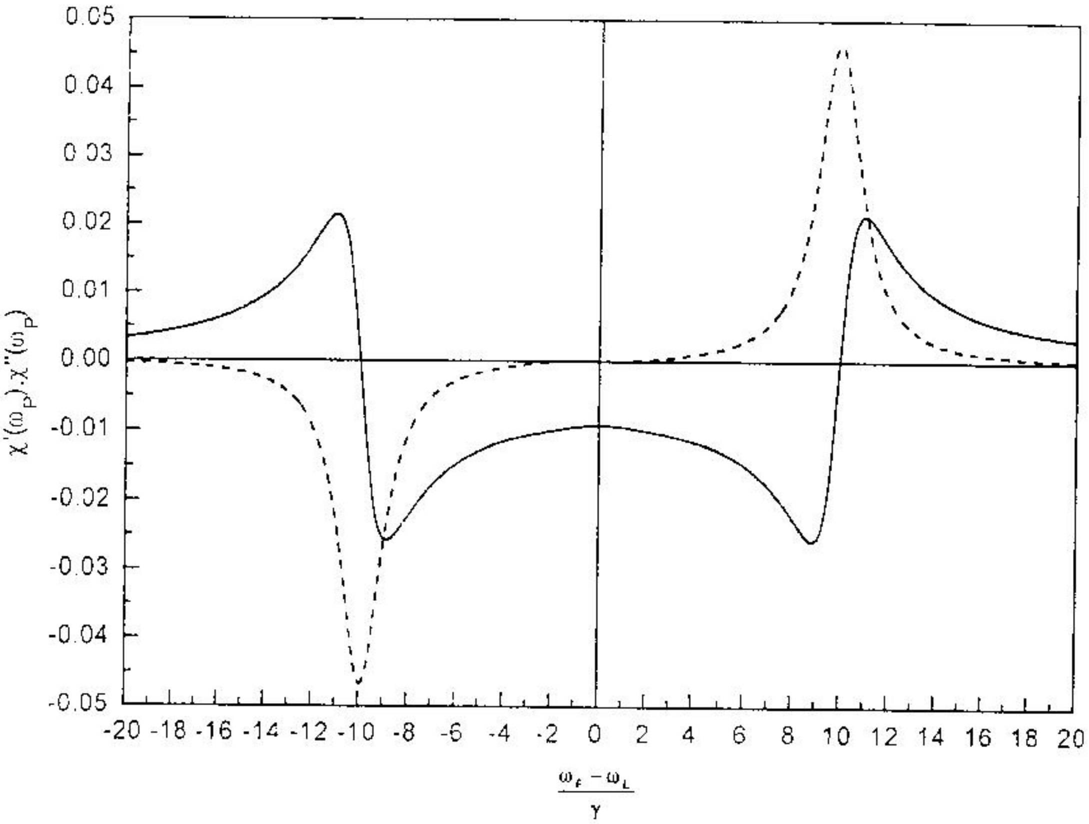

Figure 4. The same as in figure 3, but now with $\Phi = 0$.

$(\Omega/\gamma = 5, \Delta = 0)$ and for squeeze vacuum parameters $N = 0.5$ and $\Phi = \pi$. Interestingly, by changing the squeeze phase to $\Phi = 0$, the effect of enhanced refractive index without absorption is now located at the Rabi sidebands $(|(\omega_P - \omega_L)/\gamma| \simeq 2\Omega/\gamma)$ as shown in figure 4.

In conclusion, we have shown that it is possible to get a finite refractive index accompanied by zero absorption at the central frequency $(\omega_P = \omega_L)$ of a probe beam probing two-level atoms damped by a broadband squeezed vacuum and driven by a weak coherent field. This, together with the ability of the same theoretical model to exhibit a large refractive index accompanied by vanishing absorption, not only at the Rabi sidebands but also near the central frequency of the spectrum, thus offers the greater advantage in tunability. The results here are accessible to present experiments. Finally, we mention that in a related study [13] we have shown that in the absence of a coherent driving field, a system of $N_a = 3$ cooperative atoms damped by a resonant broadband squeezed vacuum produces a high refractive index accompanied by zero absorption. In this case the analysis shows that atomic coherences are generated via the squeezed vacuum field. Effects of Doppler broadening, for single-atom and $N_a \gg 1$ cooperative atom systems in intense coherent fields in both normal and squeezed vacua, on the present results are being examined and compared with the generalized dispersion relations derived for a dielectric system in squeezed vacua [14].

We acknowledge correspondence and discussions with Professor H Freedhoff and L K Teck. SSH acknowledges the hospitality of Professor R K Bullough at UMIST. SKN and MRBW acknowledge the support of Malaysia IPRA R & D 09-02-03-0337.

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