Kinetic Investigation of Consecutive-Parallel Reactions in the Non-Catalytic Process of Ethylene Oxide Hydrolysis

MARWAN M. BATIHA

Department Of Science, Jerash Private University, Jerash, Jordan

ABSTRACT. A large class of consecutive-parallel addition reactions with a large number of steps has been considered. The system of differential equations describing the kinetics of consecutive-parallel reactions in the non-catalytic process of ethylene oxide hydrolysis has been investigated. By a series of assumptions it is possible to obtain the analytical solution for the distribution of reaction products at non-zero initial conditions. The analytical solution of differential equations as a function of time has been obtained. The comparison between the solutions of the kinetics model with the reported experimental data has shown its sufficient adequacy.

KEYWORDS: consecutive-parallel reactions, ethylene oxide hydrolysis, chemical kinetics.

1. Introduction

Consecutive-parallel reactions are widely used in the technology of organic synthesis for the production of different products. The main aspect of this class of reactions is the formation of a spectrum of desired products and byproducts. This class of reactions includes: Polyoxoethylation [1], acidic polymerization of olefins [1,2], chlorination [1, 3, 4, 5, 6, 7], fluorination [8, 9], alkylation [1, 10, 11], hydrogenation [3, 12] and sulfonation [13]. The kinetics of most of these processes has been studied [3-14]. From all this class of processes, it is possible to discharge reaction of ethylene glycol synthesis [15]. The non-catalytic process of ethylene oxide (EO) hydrolysis is the most widespread way of ethylene glycol synthesis, because it is the simplest and economically justified [16]. The reactions products include: mono-, di-, tri-, tetra-, and polyethylene

glycols, which are valuable chemicals. Monoethylene glycol (MEG) is used as antifreezing liquid for different kinds of engines, as a plastifier, and as a solvent [15]. Diethylene glycol (DEG) can be used as a solvent, a plastifier and a fogger [17].

In this work, we report the kinetics of consecutive-parallel reactions using the non-catalytic process of ethylene oxide hydrolysis as a model. There are various views on the suggestions concerning the mechanism of the non-catalytic process of ethylene oxide hydrolysis reaction. So the set of differential equations describing the kinetics of this reaction is in many forms. Lebedev et al. had proposed an equation's system to calculate the quantity of the reaction's products [18,19]. Natta and Mantica had proposed another solution [20]. A similar solution was obtained by Gekhtman [3]. A more general form was obtained by Weibull and Nycander [21]. All of these forms are obtained by integrating the system of differential equations at zero initial conditions. In this work an analytical solution for distribution of reaction's products at non-zero initial conditions has been proposed.

2. Kinetic investigation of multi-step consecutive-parallel reactions

The elementary steps of the consecutive- parallel reactions are the following:

$$A + B \xrightarrow{K} P_{1}$$

$$A + P_{1} \xrightarrow{K_{1}} P_{2}$$

$$A + P_{2} \xrightarrow{K_{2}} P_{3}$$

$$A + P_{i} \xrightarrow{K_{i}} P_{i+1}$$

The kinetic equations can be described as follows:

$$\frac{dX}{d\tau} + K \cdot X [Y + m_1 \cdot X_1 + m_2 \cdot X_2 + m_3 \cdot X_3 + \cdots] = 0$$

$$\frac{dY}{d\tau} + K \cdot X \cdot Y = 0$$

$$\frac{dX_1}{d\tau} + K \cdot X \cdot [m_1 \cdot X_1 - Y] = 0$$

$$\frac{dX_2}{d\tau} + K \cdot X \cdot [m_2 \cdot X_2 - m_1 \cdot X_1] = 0$$
(1)

With initial conditions: $X|_{\tau=0} = X^0$, $Y|_{\tau=0} = Y^0$, $X_i|_{\tau=0} = X_i^0$.

where: X and Y – concentration of initial reactants A and B respectively. X_i – concentration of reaction products P_i , $m_i = \frac{K_i}{K} = const$.

If the reaction is carried out in adiabatic conditions, the heat balance equation can be written as:

$$\frac{dT}{d\tau} - \frac{1}{C_P \cdot \rho} \cdot K \cdot X \cdot [(-\Delta H) \cdot Y + (-\Delta H_1) \cdot m_1 \cdot X_1 + (-\Delta H_2) \cdot m_2 \cdot X_2 + \cdots] = 0$$
(2)

where: T - temperature of the reaction mixture, ΔH , ΔH_i - enthalpy of the reaction steps, C_p - isobaric heat capacity, ρ - density of the reaction mixture.

Let's enter a new generalized variable:

$$dt = K \cdot X \cdot d\tau \tag{3}$$

All variables are normalized relative to Y⁰:

$$\frac{X}{Y^{0}} = x_{;} \frac{Y}{Y^{0}} = y_{;} \frac{X_{i}}{Y^{0}} = x_{i;} \frac{T}{Y^{0}} = \Theta$$

We shall obtain a system of differential equations (4):

$$\frac{dx}{dt} + [y + m_1 \cdot x_1 + m_2 \cdot x_2 + m_3 \cdot x_3 + \cdots] = 0$$

$$\frac{dy}{dt} + y = 0$$

$$\frac{dx_1}{dt} + [m_1 \cdot x_1 - y] = 0$$

$$\frac{dx_2}{dt} + [m_2 \cdot x_2 - m_1 \cdot x_1] = 0$$

$$\dots$$

$$\frac{d\Theta}{dt} - \frac{1}{C_P \cdot \rho} \cdot [(-\Delta H) \cdot y + (-\Delta H_1) \cdot m_1 \cdot x_1 + (-\Delta H_2) \cdot m_2 \cdot x_2 + \cdots] = 0$$
(4)

With initial conditions:

$$x\Big|_{t=0} = x^0; y\Big|_{t=0} = 1; x_i\Big|_{t=0} = x_i^0; \Theta\Big|_{t=0} = \Theta^0$$

We have a system of linear differential equations, where the second equation in the system is homogeneous. The solution starts with the second equation of system (4), and further, equations for x_1, x_2, \cdots can be solved sequentially, then it is possible to solve these equations for x and Θ . By using a numerical integration of equation (3), taking into account the Arrhenius equation, it is possible to go back to the initial variable τ . However the analytical solution of this system in interval t is extremely tedious at a large number of considered reaction steps.

It was considered that the ratio of rate constants of all reaction steps to a rate constant of the first step are equal $[18,19,23,24^{l}]$, i.e. $m_1 = m_2 = m_3 = \cdots = m$. So the system of differential equations (4), at infinite number of reaction steps, can be written as follows:

$$\frac{dx}{dt} + \left[y + m \cdot \sum_{i=1}^{\infty} x_i\right] = 0$$

$$\frac{dy}{dt} + y = 0$$

$$\frac{dx_1}{dt} + \left[m \cdot x_1 - y\right] = 0$$

$$\frac{dx_2}{dt} + m \cdot \left[x_2 - x_1\right] = 0$$

$$\dots$$

$$\frac{d\Theta}{dt} - \frac{1}{C_P \cdot \rho} \cdot \left[\left(-\Delta H\right) \cdot y + m \cdot \sum_{i=1}^{\infty} \left(-\Delta H_i\right) \cdot x_i\right] = 0$$
By taking the sum of all equations except equations for x and Θ , we obtain

By taking the sum of all equations except equations for x and Θ , we obtain the following equation:

$$\frac{dy}{dt} + \sum_{i=1}^{\infty} \frac{dx_i}{dt} = 0$$

Integrating this equation we shall obtain:

$$\sum_{i=1}^{\infty} x_i = C^0 - y$$
(6)
where: $C^0 = 1 + \sum_{i=1}^{\infty} x_i^0$

This equation holds true for a finite number of reaction steps n if X_n is small comparing with $\sum_{i=1}^{n-1} x_i + y$. So the system of equations (5) can be written as follows:

$$\frac{dx}{dt} + \left[y \cdot (1-m) + m \cdot C^{0}\right] = 0$$

$$\frac{dy}{dt} + y = 0$$

$$\frac{dx_{1}}{dt} + \left[m \cdot x_{1} - y\right] = 0$$

$$\frac{dx_{2}}{dt} + m \cdot \left[x_{2} - x_{1}\right] = 0$$

$$\frac{d\Theta}{dt} - \frac{1}{C_{P} \cdot \rho} \cdot \left[\left(-\Delta H\right) \cdot y + m \cdot \left(C^{0} - y\right) \cdot \sum_{i=1}^{\infty} \left(-\Delta H_{i}\right)\right] = 0$$
(7)

The general solution can be presented as:

$$y = \exp(-t)$$

$$x(t) = (m-1) \cdot [1 - \exp(-t)] - m \cdot C^{0} \cdot t + x^{0}$$

$$x_{1}(t) = \frac{1}{m-1} \cdot [\exp(-t) - \exp(-m \cdot t)] + x_{1}^{0} \cdot \exp(-m \cdot t)$$

$$x_{i+1}(t) = \frac{m}{m-1} \cdot \left[x_{i}^{*}(t) - \frac{1}{m} \cdot \exp(-m \cdot t) \cdot \frac{1}{i!} \cdot (m \cdot t)^{i} \right] +$$

$$+ \exp(-m \cdot t) \cdot \left[\sum_{k=1}^{i+1} x_{k}^{0} \cdot \frac{1}{(i-k+1)!} \cdot (m \cdot t)^{i-k+1} \right]$$

$$i = 1, 2, 3, \cdots$$

(8)

where: $x_i^*(t) = x_i(t) - x_i^0 \cdot \exp(-m \cdot t)$, $i = 1, 2, 3, \cdots$ - the solution at zero initial conditions.

A similar solution at zero initial conditions on x_i had been reported [18], which is a particular case of our work. Moreover, such solution does not include calculations for reactors with recycles.

The final distribution of products can be obtained, if the left side of the second equation of the system (8) equals zero, i.e. x(t) = 0, then:

$$0 = (m-1) \cdot \left[1 - \exp(-t_f)\right] - m \cdot C^0 \cdot t_f + x^0$$
(9)

By solving equation (9) for t_f and substituting it in remaining equations of

the system (8) it is possible to obtain values for y^{∞} , x_1^{∞} , x_2^{∞} , \cdots etc.

Let's consider the goodness of the proposed solution on the non-catalytic process of EO hydrolysis as a model of consecutive-parallel reactions. The elementary steps of the non-catalytic process of EO hydrolysis were presented as follows [24]:

$$C_{2}H_{4}O + H_{2}O \xrightarrow{K} C_{2}H_{6}O_{2}$$

$$C_{2}H_{4}O + C_{2}H_{6}O_{2} \xrightarrow{K_{1}} C_{4}H_{10}O_{3}$$

$$C_{2}H_{4}O + C_{4}H_{10}O_{3} \xrightarrow{K_{2}} C_{6}H_{14}O_{4}$$

$$C_{2}H_{4}O + C_{6}H_{14}O_{4} \xrightarrow{K_{3}} C_{8}H_{18}O_{5}$$

The rate constant of the first step (K) as a function of temperature was determined by Miki et al. as $K = 2.88 \cdot 10^5 \cdot [H_2O]_0^2 \cdot \exp(-21000/RT)$ [23], where $[H_2O]_0^2$ is the initial concentration of water. An equation of the form $K = 7.123 \cdot 10^8 \cdot \exp(-21193/RT)$ was suggested [24]. The ratio of rate constants $\frac{K_1}{K}$, $\frac{K_2}{K}$ and $\frac{K_3}{K}$ were determined as 2.0, 2.0, 2.0 respectively [23] and 2.946, 3.229, 3.114 respectively [24]. In this work the ratio of rate constants are assumed to be equal to the average value, i.e. $m = \frac{K_i}{K} = 3.096$.

The adequacy of the solution we have obtained to the previously obtained experimental data [17] has been checked. Fig.1, Fig.2, and Fig.3 illustrate the comparison of experimental data with the solution of the kinetic model for MEG,



Fig.1 Distribution of monoethylene glycol output.

The enthalpies of all reaction steps are assumed to be equal [17], i.e. $(-\Delta H_1) \cong (-\Delta H_2) \cong \cdots$

Then the equation of heat balance can be written as:

$$\frac{d\Theta}{dt} - \alpha \cdot \left[(1 - m) \cdot y + m \cdot C^0 \right] = 0$$
(10)
Where: $\alpha = \frac{(-\Delta H)}{C_P \cdot \rho}$

By comparing equation (10) with the first equation of system (7), the



temperature can be determined as:



Fig. 3 Distribution of triethylene glycol output.

$$\frac{d\Theta}{dt} = -\alpha \cdot \frac{dx}{dt} \quad \text{, or} \qquad \Theta = \Theta^0 + \alpha \cdot \left(x^o - x\right) \tag{11}$$

Consider now some particular cases.

I) Assume, that the component B is in considerable excess, i.e. $y = y_0 = 1$. The system of differential equations (5) can be written as:

$$\frac{dx}{dt} + \left[1 + m \cdot \sum_{i=1}^{\infty} x_i\right] = 0$$

$$\frac{dx_1}{dt} + m_1 \cdot x_1 = 1$$

$$\frac{dx_2}{dt} + m \cdot x_2 = m \cdot x_1$$

$$\frac{dx_3}{dt} + m \cdot x_3 = m \cdot x_2$$

$$\dots$$

$$\frac{d\Theta}{dt} - \alpha \cdot \left[1 + m \cdot \sum_{i=1}^{\infty} x_i\right] = 0$$
(12)

By taking the sum of equations for x_1, x_2 ...etc, we obtain the following equation:

$$\sum_{i=1}^{\infty} \frac{dx_i}{dt} = 1, \text{ or } \sum_{i=1}^{\infty} x_i = t + \sum_{i=1}^{\infty} x_i^0$$
(13)

The solution can be presented as:

$$x_{1} = \frac{1}{m} \cdot [1 - \exp(-m \cdot t)] + x_{1}^{0} \cdot \exp(-m \cdot t)$$

$$x_{i+1}(t) = \frac{1}{m} \cdot \left[x_{i}^{*}(t) - \frac{1}{i!} \cdot (m \cdot t)^{i} \right] +$$

$$+ \exp(-m \cdot t) \cdot \left[\sum_{k=1}^{i+1} x_{k}^{0} \cdot \frac{1}{(i-k+1)!} \cdot (m \cdot t)^{i-k+1} \right]$$
(14)

In view of the system of equations (14), equation for x can be written as:

$$\frac{dx}{dt} + C_1^0 + m \cdot t$$
, where: $C_1^0 = 1 + m \cdot \sum_{i=1}^{\infty} x_i^0$

The solution can be presented as:

$$x = x^{0} - C_{1}^{0} \cdot t - \frac{m \cdot t^{2}}{2}$$
(15)

It is possible to find t_f from equation (15), if x = 0:

$$t_f = \frac{-C_1^0 + \sqrt{(C_1^0)^2 + 2 \cdot m \cdot x^0}}{m} = \frac{-C_1^0 + \sqrt{\Delta}}{m}$$

In view of equation's system (14), the solution of equation for Θ can be written as:

$$\Theta = \alpha \cdot C^0 \cdot t + \frac{\alpha \cdot m \cdot t^2}{2} + \Theta^0$$
(16)

II) Assume, that the component A is in excess, i.e. $x=x^0=$ const. A system of equations can be presented as:

$$\frac{dy}{dt} + y = 0$$

$$\frac{dx_1}{dt} + m \cdot x_1 = y$$

$$\frac{dx_2}{dt} + m \cdot x_2 = m \cdot x_1$$
(17)

The solution for y and x_i can be written as the general case (8). Equation for temperature can be presented as:

$$\frac{d\Theta}{dt} - \alpha \cdot \left[(1 - m) \cdot y + m \cdot C^0 \right] = 0$$

Assume, that $y = \exp(-t)$, the solution can be written as:

$$\Theta = -\alpha \cdot \left[(1 - m) \cdot (1 - \exp(-t)) + m \cdot C^0 \cdot t \right] + \Theta^0$$
(18)

3. Conclusions

The system of differential equations describing the kinetics of consecutiveparallel addition reactions with a large number of steps has been investigated. The analytical solution for distribution of reaction products at non-zero initial conditions has been obtained. The predictions obtained with the solutions of the kinetic model were compared with the experimental data earlier obtained by Shargarodski [17]. It has been shown that, the solutions of the kinetic model are adequately describing the given process. Particular cases of reaction realization in conditions of excess of one of initial reactants have been considered. The solutions for these cases have been obtained.

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Notation

- $\begin{array}{c} C_p \\ T \end{array}$ isobaric heat capacity, J/ kg K
- temperature of the reaction mixture, K
- Х concentration of initial reactant A, mol/l
- concentration of reaction products P_i , mol/l X_i
- x_i^* the solution at zero initial conditions
- Y concentration of initial reactant B, mol/l

Greek letters

 ΔH , ΔH_i enthalpy of the reaction steps, kJ/mol

density of the reaction mixture, kg/m³ ρ

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